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Charnley AK (1992). Mechanisms of fungal pathogenesis in insects with particular reference to locusts. In: Lomer CJ, Prior C (eds) *Biological Controls of Locusts and Grasshoppers: Proceedings of an international workshop held at Cotonou, Benin.* Oxford: CAB International, pp 181-190.

Mundree SG, Farrant JM (2000). Some physiological and molecular insights into the mechanisms of desiccation tolerance in the resurrection plant *Xerophyta viscata* Baker. In Cherry et al. (eds) *Plant tolerance to abiotic stresses in Agriculture: Role of Genetic Engineering*, Kluwer Academic Publishers, Netherlands, pp 201-222.

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Full Length Research Paper

Characterizations of semi-groups that are semi-lattices of left groups

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Zadeh in 1965 introduce the concept of a fuzzy set which now have a wide range of applications in various fields of Engineering Sciences, Computer Science and Management Sciences. It is worth mentioning here that for applications of fuzzy sets mostly associative algebraic structures are used such as in 2003, Mordeson, Malik and Kuroki have discovered the applications of fuzzy semigroups in fuzzy coding, fuzzy finite-state machines and fuzzy languages. We have applied fuzzy set theory to associative algebraic structures and have discussed related properties. Specifically we have characterized semigroups which are semilattices of left groups by the properties of generalized fuzzy ideals.

Key words: $(\in, \in \vee q_k)$ -fuzzy ideals, $(\in, \in \vee q_k)$ -fuzzy bi-ideals, $(\in, \in \vee q_k)$ -fuzzy quasi ideals.

INTRODUCTION

Fuzzy set theory (Zadeh, 1965) and applications in several branches of Science are growing day by day. Fuzzy set theory on semi-groups has already been developed. Murali (2004) defined the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on a fuzzy subset. The idea of quasi-coincidence of a fuzzy point with a fuzzy set is defined in the study of Pu and Liu. (1980). It is worth pointing out that Bhakat and Das (1992) gave the concept of (α, β) -fuzzy sub-groups by using the concept of belongingness \in and quasi-coincident with relation q between a fuzzy point and a fuzzy subgroup, and introduced the concept of an $(\in, \in \vee q)$ -fuzzy subgroups, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$. Davvaz defined $(\in, \in \vee q_k)$ -fuzzy subnearings and ideals of a near ring in (Davvaz, 2006). Jun and Song (2006) initiated the study of (α, β) -fuzzy interior ideals of a semi-group. In the study of Shabir et al. (2010), regular semi-groups

were characterized by the properties of $(\in, \in \vee q_k)$ -fuzzy ideals.

A non-empty sub-set A of a semi-group S is called a sub-semi-group of S if $A^2 \subseteq A$. A non-empty subset J of S is called a left (right) ideal of S if $SJ \subseteq I$ ($JS \subseteq I$). J is called a two-sided ideal or simply an ideal of S if it is both left and right ideal of S . A non-empty sub-set Q of S is called a quasi-ideal of S if $QS \cap SQ \subseteq Q$. A non-empty sub-set B of S is called a generalized bi-ideal of S if $BSB \subseteq B$. A non-empty sub-set B of S is called a bi-ideal of S if it is both a sub-semi-group and a generalized bi-ideal of S . A sub-semi-group I of S is called an interior ideal of S if $SIS \subseteq I$. Let f and g be any fuzzy sub-sets of a semi-group S , then the product $f \circ g$ is defined by:

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$$(f \circ g)(a) = \begin{cases} \bigvee_{a=bc} \{f(b) \wedge g(c)\}, & \text{if there exist } a, b \in S, \text{ such that } a = bc \\ 0, & \text{otherwise.} \end{cases}$$

A fuzzy sub-set f of a semi-group S is called a fuzzy sub-semi-group of S if $f(xy) \geq f(x) \wedge f(y)$ for all $x, y \in S$. A fuzzy subset f of a semi-group S is called fuzzy left (right) ideal of S if $f(xy) \geq f(y)$ ($f(xy) \geq f(x)$) for all $x, y \in S$. A fuzzy sub-set f of a semi-group S is called fuzzy two-sided ideal of S if it is both fuzzy left and fuzzy right ideal of S .

A fuzzy sub-semi-group f of a semi-group S is called fuzzy bi-ideal of S if $f(xyz) \geq f(x) \wedge f(z)$, for all x, y and $z \in S$. A fuzzy sub-set f of a semi-group S is called fuzzy generalized bi-ideal of S if $f(xyz) \geq f(x) \wedge f(z)$, for all x, y and $z \in S$. A fuzzy sub-semi-group f of a semi-group S is called fuzzy interior ideal if $f(xaz) \geq f(a)$, for all x, a and $y \in S$. A fuzzy sub-set f of a semi-group S is called fuzzy quasi-ideal of S if $f(x) \geq (f \circ S)(x) \wedge (S \circ f)(x)$, for all $x \in S$. An element a in S is called regular if there exists an element x in S such that $a = axa$. S is called regular if every element of S is regular. It is well known in semi-group theory that every one sided ideal of S is quasi-ideal of S and every quasi-ideal of S is bi-ideal of S .

Definition 1

For a fuzzy sub-set δ of a semi-group S and $t \in (0, 1]$, the crisp set $U(\delta; t) = \{x \in S \text{ such that } \delta(x) \geq t\}$ is called level subset of δ .

Definition 2

A fuzzy sub-set δ of a semi-group S is of the form:

$$\delta(y) = \begin{cases} t \in (0, 1] & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

This form is said to be a fuzzy point with support x and value t and is denoted by x_t (Mordeson et al., 2003).

A fuzzy point x_t is said to belong to (resp. quasi-coincident with) a fuzzy set δ , written as $x_t \in \delta$ (resp. $x_t q \delta$) if $\delta(x) \geq t$ (resp. $\delta(x) + t > 1$). If $x_t \in \delta$ or

(resp. and) $x_t q \delta$, then, we write $x_t \in \overline{\vee q}(\in \wedge q) \delta$.

The symbol $\overline{\vee q}$ means $\in \vee q$ does not hold. For any two fuzzy sub-sets, δ and g of S , $\delta \leq g$ means that, for all $x \in S$, $\delta(x) \leq g(x)$. Generalizing the concept of $x_t q \delta$, Jun et al. (2006) defined:

$$x_t q_k \delta$$

Where $k \in [0, 1)$, as $\delta(x) + t + k > 1$. $x_t \in \vee q_k \delta$ if $x_t \in \delta$ or $x_t q_k \delta$. Let f and g be any two fuzzy subsets of an AG-groupoid S , then, for $k \in [0, 1)$, the product $f \circ_k g$ is defined by:

$$(f \circ_k g)(a) = \begin{cases} \bigvee_{a=bc} \{f(b) \wedge g(c) \wedge \frac{1-k}{2}\}, & \text{if there exist } b, c \in S, \text{ such that } a = bc. \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3

A fuzzy sub-set δ of a semi-group S is called an $(\in, \in \vee q_k)$ -fuzzy sub-semi-group of S if for all $x, y \in S$ and $t, r \in (0, 1]$, it satisfies, $x_t \in \delta, y_r \in \delta$ implies that $(xy)_{\min\{t, r\}} \in \vee q_k \delta$ (Shabir et al., 2010a).

Definition 4

A fuzzy subset δ of a semi group S is called an $(\in, \in \vee q_k)$ -fuzzy left (right) ideal of S if for all $x, y \in S$ and $t, r \in (0, 1]$, it satisfies, $x_t \in \delta$ implies $(yx)_t \in \vee q_k \delta$ ($x_t \in \delta$ implies $(xy)_t \in \vee q_k \delta$) (Shabir et al., 2010a).

Definition 5

A fuzzy sub-semi-group f of a semi-group S is called an $(\in, \in \vee q_k)$ -fuzzy interior ideal of S if for all $x, y, z \in S$ and $t, r \in (0, 1]$ the following condition holds: $y_t \in f$ implies $((xy)z)_t \in \vee q_k f$ (Shabir et al., 2010a).

Definition 6

A fuzzy subset f of a semi-group S is called an $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal of S if $x_t \in f$ and

$z_r \in S$ implies $((xy)z)_{\min\{t,r\}} \in \vee q_k f$, for all $x, y, z \in S$ and $t, r \in (0,1]$ (Shabir et al., 2010a).

Definition 7

A fuzzy subset f of a semi-group S is called a $(\in, \in \vee q_k)$ -fuzzy bi-ideal of S if for all $x, y, z \in S$ and $t, r \in (0,1]$ the following conditions holds (Shabir et al., 2010a):

- (i) If $x_t \in f$ and $y_r \in S$ implies $(xy)_{\min\{t,r\}} \in \vee q_k f$,
- (ii) If $x_t \in f$ and $z_r \in S$ implies $((xy)z)_{\min\{t,r\}} \in \vee q_k f$.

Theorem 1

A fuzzy subset δ of a semi-group S is called a $(\in, \in \vee q_k)$ -fuzzy left (right) ideal of S if and only if $\delta(xy) \geq \min\{\delta(y), \frac{1-k}{2}\} (\delta(xy) \geq \min\{\delta(x), \frac{1-k}{2}\})$ (Shabir et al., 2010a).

Theorem 2

A fuzzy subset δ of a semi-group S is called a $(\in, \in \vee q_k)$ -fuzzy bi-ideal of S if and only if (Shabir et al., 2010a):

- (i) $(\forall x, y \in S \text{ and } k \in [0,1)) \delta(xy) \geq \min\{\delta(x), \delta(y), \frac{1-k}{2}\}$
- (ii) $(\forall x, y, z \in S \text{ and } k \in [0,1)) \delta((xy)z) \geq \min\{\delta(x), \delta(z), \frac{1-k}{2}\}$.

Theorem 3

A fuzzy subset δ of a semi-group S is called an $(\in, \in \vee q_k)$ -fuzzy interior ideal of S if and if $(\forall x, y, z \in S \text{ and } k \in [0,1)) \delta((xy)z) \geq \min\{\delta(y), \frac{1-k}{2}\}$ (Shabir et al., 2010a).

Definition 8

A fuzzy subset δ of a semi-group S is called an $(\in, \in \vee q_k)$ -fuzzy quasi-ideal of S , if it satisfies:

$$f(x) \geq \min\{(f \circ I)(x), (I \circ f)(y), \frac{1-k}{2}\}.$$

Where I is the fuzzy subset of S mapping every

element of S on 1 (Shabir et al., 2010b).

A semi-group S is said to be left simple if it contains no proper left ideal.

Definition 9

Let f and g be fuzzy subsets of S . We define the fuzzy subsets $f_k, f \wedge_k g$ and $f \circ_k g$ of S as follows:

- (i) $f_k(a) = f(a) \wedge \frac{1-k}{2}$.
- (ii) $(f \wedge_k g)(a) = (f \wedge g)(a) \wedge \frac{1-k}{2}$.
- (iii) $(f \circ_k g)(a) = (f \circ g)(a) \wedge \frac{1-k}{2}$, for all $a \in S$.

Lemma 1

Let f and g be fuzzy subsets of S . Then following conditions hold (Shabir et al., 2010a):

- (i) $(f \wedge_k g) = (f_k \wedge g_k)$.
- (ii) $(f \circ_k g) = (f_k \circ g_k)$.

Definition 10

Let A be any sub-set of a semi-group S , then characteristic function $(C_A)_k$ is defined as:

$$(C_A)_k(a) = \begin{cases} \geq \frac{1-k}{2} & \text{if } a \in A \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 2

Let A and B be any non-empty subsets of a semi-group S . Then, the following conditions hold (Shabir et al., 2010a):

- (i) $(C_A \wedge_k C_B) = (C_{A \cap B})_k$.
- (ii) $(C_A \circ_k C_B) = (C_{AB})_k$.

Example 1

Let $S = \{1, 2, 3\}$ be a semi-group with binary operation " \otimes ", as defined in the following Cayley table:

·	1	2	3
1	1	1	1
2	1	2	3
3	3	3	3

Let us define a fuzzy subset δ of S , as:

$$\delta(1) = 0.8, \delta(2) = 0.7, \delta(3) = 0.6.$$

Then clearly δ is a $(\in, \in \vee q_k)$ -fuzzy ideal of S .

SEMI-GROUPS WHICH ARE SEMI-LATTICES OF LEFT GROUPS

Lemma 3

Let Q be a non-empty subset of a semi-group S . Then, Q is a quasi-ideal of S if and only if $(C_Q)_k$ is a $(\in, \in \vee q_k)$ -fuzzy quasi-ideal of S (Shabir et al., 2010a).

Lemma 4

A non-empty subset L of a semi-group S is left (right) ideal of S if and only if $(C_L)_k$ is an $(\in, \in \vee q_k)$ -fuzzy left (right) ideal of S (Shabir et al., 2010a).

Lemma 5

A non-empty subset B of a semi-group S is a bi(interior) ideal of S if and only if $(C_B)_k$ is an $(\in, \in \vee q_k)$ -fuzzy bi(interior) ideal of S (Shabir et al., 2010a).

Theorem 4

For a semi-group S , the following conditions are equivalent (Petrich, 1973):

- (i) S is a semi-lattice of left groups.
- (ii) S is regular and $aS \subseteq Sa$ for every $a \in S$.

Theorem 5

Let S be a semi-group that is semi-lattice of left groups.

Then, the following conditions hold:

- (i) Every $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal of S is an $(\in, \in \vee q_k)$ -fuzzy right ideal of S .
- (ii) Every $(\in, \in \vee q_k)$ -fuzzy interior ideal of S is an $(\in, \in \vee q_k)$ -fuzzy left ideal of S .

Proof

(i) Let f be any $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal of S , and $a, b \in S$. Then, by Theorem 4, there exist elements $x, y \in S$ such that $a = axa$ and $ab = ya$. Thus, we have:

$$ab = (axa)b = (ax)(ab) = (ax)(ya) = a(xy)a.$$

Since f is $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal of S , we have:

$$f(ab) = f(a(xy)a) \geq f(a) \wedge f(a) \wedge \frac{1-k}{2} = f(a) \wedge \frac{1-k}{2}.$$

Therefore, f is an $(\in, \in \vee q_k)$ -fuzzy right ideal of S .

ii. Let g be any $(\in, \in \vee q_k)$ -fuzzy interior ideal of S , and $a, b \in S$. Then, by Theorem 4, there exists $z \in S$ such that $b = bzb$. Since g is an $(\in, \in \vee q_k)$ -fuzzy interior ideal of S , we have:

$$g(ab) = g(a(bzb)) = g(ab(zb)) \geq g(b) \wedge \frac{1-k}{2}.$$

Thus, g is a $(\in, \in \vee q_k)$ -fuzzy left ideal of S .

Theorem 6

For a semi-group S , the following conditions are equivalent and

- (i) S is regular (Shabir et al., 2010a):
- (ii) $f \wedge_k g \leq f \circ_k g$ for every $(\in, \in \vee q_k)$ -fuzzy quasi-ideal f and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iii) $f \wedge_k g \leq f \circ_k g$ for every $(\in, \in \vee q_k)$ -fuzzy bi-ideal f and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .

iv $f \wedge_k g \leq f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy generalized bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy left ideal g of S .

Theorem 7

For a semi-group S , the following conditions are equivalent:

- (i) S is a semi-lattice of left groups.
- (ii) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy quasi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iii) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy quasi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy two sided ideal g of S .
- (iv) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy quasi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy interior ideal g of S .
- (v) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy left ideal g of S .
- (vi) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy two sided ideal g of S .
- (vii) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy interior ideal g of S .
- (viii) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy generalized bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy left ideal g of S .
- (ix) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy generalized bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy two sided ideal g of S .
- (x) $f \wedge_k g = f \circ_k g$ for every $(\epsilon, \in \vee q_k)$ -fuzzy generalized bi-ideal f and every $(\epsilon, \in \vee q_k)$ -fuzzy interior ideal g of S .

Proof

First assume that (i) holds. Let f and g be any $(\epsilon, \in \vee q_k)$ -fuzzy generalized bi and any $(\epsilon, \in \vee q_k)$ -fuzzy interior ideals of S , respectively. Let $a \in S$, then, by Theorem 4, S is regular, so there exists an element $x \in S$, such that $a = axa = axaxa$. Since g is an

$(\epsilon, \in \vee q_k)$ -fuzzy interior ideal g of S , so we have:

$$\begin{aligned} (f \circ_k g)(a) &= \left(\bigvee_{a=bc} \{f(b) \wedge g(c)\} \right) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge g(xa(xa)) \wedge \frac{1-k}{2} \\ &\geq f(a) \wedge g(a) \wedge \frac{1-k}{2} = (f \wedge_k g)(a). \end{aligned}$$

Therefore, $f \wedge_k g \leq f \circ_k g$. Now, by Theorem 5, f is an $(\epsilon, \in \vee q_k)$ -fuzzy right ideal and g is an $(\epsilon, \in \vee q_k)$ -fuzzy left ideal of S . Thus we have:

$$\begin{aligned} (f \circ_k g)(a) &= \left(\bigvee_{a=bc} \{f(b) \wedge g(c)\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ f(b) \wedge \frac{1-k}{2} \wedge g(c) \wedge \frac{1-k}{2} \right\} \right) \\ &\leq \left(\bigvee_{a=bc} \{f(bc) \wedge g(bc)\} \right) \wedge \frac{1-k}{2} \\ &= f(a) \wedge g(a) \wedge \frac{1-k}{2} = (f \wedge_k g)(a). \end{aligned}$$

Therefore, $f \wedge_k g \geq f \circ_k g$. Hence, $f \wedge_k g = f \circ_k g$. Assuming that (ii) holds, then, by Theorem 6, S is regular. Let Q be any quasi-ideal of S . Then, by Lemma 3, the characteristic $(C_Q)_k$ of Q is a $(\epsilon, \in \vee q_k)$ -fuzzy quasi-ideal of S . Since S itself is a $(\epsilon, \in \vee q_k)$ -fuzzy left ideal of S , then by assumption we have:

$$\begin{aligned} ((C_Q)_k \circ_k S)(a) &= ((C_Q)_k \wedge_k S)(a) = ((C_Q)_k \wedge S)(a) \wedge \frac{1-k}{2} \\ &= (C_Q)_k(a) \wedge \frac{1-k}{2} = (C_Q)_k(a). \end{aligned}$$

Hence, $(C_Q)_k$ is $(\epsilon, \in \vee q_k)$ -fuzzy right ideal of S . Hence, by Lemma 4, Q is right ideal of S . Thus, any quasi-ideal of S is right ideal of S . Let $a \in S$, then quasi-ideal Sa of S is a right ideal, and since S is regular, so we have:

$$aS \subseteq (aSa)S = ((aS)a)S \subseteq (Sa)S \subseteq Sa.$$

Therefore, by Theorem 4, S is semi-lattice of left groups.

Theorem 8

For a semi-group S , the following conditions are

equivalent (Shabir et al., 2010a):

- (i) S is regular
- (ii) $f_k = f \circ_k S \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal f of S .
- (iii) $f_k = f \circ_k S \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy bi-ideal f of S .
- (iv) $f_k = f \circ_k S \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy quasi-ideal f of S .

Theorem 9

For a semi-group S , the following conditions are equivalent:

- (i) S is a semi lattice of left groups.
- (ii) $f \wedge_k g = f \circ_k g \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy quasi-ideal f and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iii) $f \wedge_k g = f \circ_k g \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy bi-

- ideal f and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iv) $f \wedge_k g = f \circ_k g \circ_k f$ for every $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal f and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .

Proof

First assume that (i) holds, then by Theorem 4, S is regular, and for $a \in S$ there exist elements $x, y, z \in S$, such that $a = axa = axaxa$ and $ay = za$. Let f and g be any $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal and any $(\in, \in \vee q_k)$ -fuzzy left ideal of S , respectively. Then we have:

$$g(xay) \geq g(ay) \wedge \frac{1-k}{2} = g(za) \wedge \frac{1-k}{2} \geq g(a) \wedge \frac{1-k}{2}.$$

Therefore, g is a $(\in, \in \vee q_k)$ -fuzzy interior ideal of S . Now we have:

$$\begin{aligned} (f \circ_k g \circ_k f)(a) &= (f \circ g \circ f)(a) \wedge \frac{1-k}{2} \leq (S \circ g \circ S)(a) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \{(f \circ S)(b) \wedge f(c)\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{S(p) \wedge g(q)\} \wedge S(c) \right\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{1 \wedge g(q)\} \wedge 1 \right\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} g(q) \right\} \right) \wedge \frac{1-k}{2} \\ &\leq \bigvee_{a=(pq)c} \left\{ g(pqc) \wedge \frac{1-k}{2} \right\} \\ &= g(a) \wedge \frac{1-k}{2} = g_k(a). \end{aligned}$$

Thus, $(f \circ_k g \circ_k f)(a) \leq g_k(a)$. Also:

$$\begin{aligned} (f \circ_k g \circ_k f)(a) &= (f \circ g \circ f)(a) \wedge \frac{1-k}{2} \leq (f \circ S \circ f)(a) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \{(f \circ S)(b) \wedge f(c)\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{f(p) \wedge S(q)\} \wedge f(c) \right\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{f(p) \wedge 1\} \wedge f(c) \right\} \right) \wedge \frac{1-k}{2} \end{aligned}$$

$$\begin{aligned}
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{f(p) \wedge f(c)\} \right\} \right) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{f(p) \wedge f(c)\} \wedge \frac{1-k}{2} \right\} \\
 &= \bigvee_{a=(pq)c} \left\{ f(p) \wedge f(c) \wedge \frac{1-k}{2} \right\} \\
 &\leq \bigvee_{a=(pq)c} \left\{ f(pqc) \wedge \frac{1-k}{2} \right\} \\
 &= f(a) \wedge \frac{1-k}{2} = f_k(a).
 \end{aligned}$$

Thus, $(f \circ_k g \circ_k f)(a) \leq f_k(a) \wedge g_k(a)$ and by Lemma 1, $(f \circ_k g \circ_k f)(a) = (f \wedge_k g)(a)$. Now, let $a \in S$ then, by Theorem 4, there exist $x, y \in S$ such that $a = axa$ and $ax = ya$. Then:

$$ax = axax = axax(ya) = (axa)(xya).$$

Thus, we have:

$$\begin{aligned}
 (f \circ_k g \circ_k f)(a) &= (f \circ g \circ f)(a) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=bc} \{ (f \circ g)(b) \wedge f(c) \} \wedge \frac{1-k}{2} \\
 &\geq \{ (f \circ g)(ax) \wedge f(a) \} \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{ax=pq} \{ f(p) \wedge g(q) \} \wedge f(a) \right) \wedge \frac{1-k}{2} \\
 &\geq f(axa) \wedge g(xya) \wedge f(a) \wedge \frac{1-k}{2} \\
 &\geq f(a) \wedge g(a) \wedge \frac{1-k}{2} = (f \wedge_k g)(a).
 \end{aligned}$$

Therefore, $(f \circ_k g \circ_k f)(a) \geq (f \wedge_k g)(a)$. Hence, $(f \circ_k g \circ_k f)(a) = (f \wedge_k g)(a)$.

It is clear that $(iv) \Rightarrow (iii) \Rightarrow (ii)$. Now assume that (ii) holds. Let f be any $(\in, \in \vee q_k)$ -fuzzy quasi-ideal of S . Then, S itself is a $(\in, \in \vee q_k)$ -fuzzy left ideal of S and so by assumption we have:

$$(f \circ_k S \circ_k f)(a) = (f \wedge_k S)(a) = (f \wedge S)(a) \wedge \frac{1-k}{2} = f_k(a).$$

This follows from Theorem 8 that S is regular. Let g be $(\in, \in \vee q_k)$ -fuzzy left ideal of S , and S itself a

$(\in, \in \vee q_k)$ -fuzzy quasi-ideal of S , so we have by assumption:

$$(S \circ_k g \circ_k S)(a) = (S \wedge_k g)(a) = (S \wedge g)(a) \wedge \frac{1-k}{2} = g_k(a).$$

Thus, g is an $(\in, \in \vee q_k)$ -fuzzy interior ideal of S . Since S is regular, therefore, g is $(\in, \in \vee q_k)$ -fuzzy two sided ideal of S . Therefore, every $(\in, \in \vee q_k)$ -fuzzy left ideal is a $(\in, \in \vee q_k)$ -fuzzy two sided ideal of S . Since S is regular and the left ideal Sa is right ideal of S , we have:

$$aS \subseteq (aSa)S = a((Sa)S) \subseteq a(Sa) = (aS)a \subseteq Sa.$$

Therefore, by Theorem 4, S is semi lattice of left groups.

Theorem 10

For a semi-group S , the following conditions are equivalent:

- (i) S is a semi lattice of left groups.
- (ii) $f \wedge_k g = f \circ_k S \circ_k g$ for every $(\in, \in \vee q_k)$ -fuzzy quasi-ideal f of S and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iii) $f \wedge_k g = f \circ_k S \circ_k g$ for every $(\in, \in \vee q_k)$ -fuzzy bi-ideal f of S and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .
- (iv) $f \wedge_k g = f \circ_k S \circ_k g$ for every $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal f of S and every $(\in, \in \vee q_k)$ -fuzzy left ideal g of S .

Proof

First assume that (i) holds, then, by Theorem 4, S is regular, and for $a \in S$ there exists element $x \in S$, such that $a = axa$. Let f and g be any $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal and any $(\in, \in \vee q_k)$ -fuzzy left ideal of S , respectively. Then by Theorem 5, every $(\in, \in \vee q_k)$ -fuzzy generalized bi-ideal f of S is $(\in, \in \vee q_k)$ -fuzzy right ideal of S . Then we have:

$$\begin{aligned}
 (f \circ_k S \circ_k g)(a) &= (f \circ S \circ g)(a) \wedge \frac{1-k}{2} \leq (S \circ S \circ g)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{ (S \circ S)(b) \wedge g(c) \} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{ S(p) \wedge S(q) \} \wedge g(c) \right\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} 1 \wedge g(c) \right\} \right) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=pqc} \left(g(c) \wedge \frac{1-k}{2} \right) \\
 &\leq \bigvee_{a=pqc} \left(g(pqc) \wedge \frac{1-k}{2} \right) \\
 &= g(a) \wedge \frac{1-k}{2} = g_k(a).
 \end{aligned}$$

Therefore, $(f \circ_k S \circ_k g)(a) \leq g_k(a)$. Also:

$$\begin{aligned}
 (f \circ_k S \circ_k g)(a) &= (f \circ S \circ g)(a) \wedge \frac{1-k}{2} \leq (f \circ S \circ S)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{ f(b) \wedge (S \circ S)(c) \} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ f(b) \wedge \left(\bigvee_{c=pq} \{ S(p) \wedge S(q) \} \right) \right\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{c=pq} \{ f(b) \wedge (1) \} \right\} \right) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=bpq} \left\{ f(b) \wedge \frac{1-k}{2} \right\} \\
 &\leq \bigvee_{a=bpq} \left\{ f(bpq) \wedge \frac{1-k}{2} \right\} \\
 &= f(a) \wedge \frac{1-k}{2} = f_k(a).
 \end{aligned}$$

Thus, $(f \circ_k S \circ_k g)(a) \leq f_k(a)$. Therefore, by Lemma 1, we have:

$$(f \circ_k S \circ_k g)(a) \leq f_k(a) \wedge g_k(a) = (f \wedge_k g)(a).$$

Now, let $a \in S$ then by Theorem 4, S is regular so there exists $x \in S$ such that $a = axa$. Then we have:

$$\begin{aligned}
 (f \circ_k S \circ_k g)(a) &= (f \circ S \circ g)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{ (f \circ S)(b) \wedge g(c) \} \right) \wedge \frac{1-k}{2} \\
 &\geq (f \circ S)(ax) \wedge g(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{ax=pq} \{ f(a) \wedge S(q) \} \right) \wedge g(a) \wedge \frac{1-k}{2} \\
 &\geq f(a) \wedge S(x) \wedge g(a) \wedge \frac{1-k}{2} \\
 &= f(a) \wedge g(a) \wedge \frac{1-k}{2} = (f \wedge_k g)(a).
 \end{aligned}$$

Therefore, $(f \circ_k S \circ_k g)(a) \geq (f \wedge_k g)(a)$.

Hence, $(f \circ_k S \circ_k g) = (f \wedge_k g)$. It is clear that:

$$(iv) \Rightarrow (iii) \Rightarrow (ii).$$

Assuming that (ii) holds. Let f and g be any $(\in, \in \vee q_k)$ -fuzzy quasi-ideal and $(\in, \in \vee q_k)$ -fuzzy left ideal of S , respectively. Then, by assumption, we have:

$$\begin{aligned}
 (f \wedge_k g) &= (f \circ_k S \circ_k g) = (f \circ S \circ g) \wedge \frac{1-k}{2} \\
 &= (f \circ (S \circ g)) \wedge \frac{1-k}{2} \subseteq (f \circ g) \wedge \frac{1-k}{2} \\
 &= (f \circ_k g).
 \end{aligned}$$

Thus, by Theorem 6, S is regular. Let g be any $(\in, \in \vee q_k)$ -fuzzy left ideal then g is $(\in, \in \vee q_k)$ -fuzzy quasi-ideal of S . Then, since S itself is an $(\in, \in \vee q_k)$ -fuzzy left ideal of S , by assumption we have:

$$(g \circ_k S \circ_k S) = (g \wedge_k S) = (g \wedge S) \wedge \frac{1-k}{2} = g_k.$$

Let L be any left ideal of S , and $a \in L$. Then by Lemma 4 $(C_L)_k$ is an $(\in, \in \vee q_k)$ -fuzzy left ideal of S . Thus, by using Lemma 2, we have:

$$(C_{LSS})_k(a) = (C_L \circ_k C_S \circ_k C_S)(a) = (C_L)_k(a) \geq \frac{1-k}{2}.$$

Therefore, $a \in LSS$. Conversely, assume that $a \in LSS$.

Then:

$$(C_L)_k(a) = (C_L \circ_k C_S \circ_k C_S)(a) = (C_{LSS})_k(a) \geq \frac{1-k}{2}.$$

Therefore, $a \in L$. Hence, $L = LSS$. Since Sa is a left ideal of a regular semi group S , then we have:

$$aS \subseteq aSaSaS = a(Sa)(SSS) \subseteq a((Sa)SS) = a(Sa) = (aS)a \subseteq Sa.$$

Therefore, by Theorem 4, S is semi lattice of left groups.

Theorem 11

For a semi group S , the following conditions are equivalent:

- (i) S is a semi lattice of left groups.
- (ii) $f \wedge_k h \wedge_k g = f \circ_k h \circ_k g$ for every $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-ideal f , every $(\epsilon, \epsilon \vee q_k)$ -fuzzy two sided ideal h

and every $(\epsilon, \epsilon \vee q_k)$ -fuzzy left ideal g of S .

(iii) $f \wedge_k h \wedge_k g = f \circ_k h \circ_k g$ for every $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-ideal f , every $(\epsilon, \epsilon \vee q_k)$ -fuzzy two sided ideal h and every $(\epsilon, \epsilon \vee q_k)$ -fuzzy left ideal g of S .

(iv) $f \wedge_k h \wedge_k g = f \circ_k h \circ_k g$ for every $(\epsilon, \epsilon \vee q_k)$ -fuzzy generalized bi-ideal f , every $(\epsilon, \epsilon \vee q_k)$ -fuzzy two sided ideal h and every $(\epsilon, \epsilon \vee q_k)$ -fuzzy left ideal g of S .

Proof

First assume that (i) holds. Let f be any $(\epsilon, \epsilon \vee q_k)$ -fuzzy generalized bi-ideal, h be any $(\epsilon, \epsilon \vee q_k)$ -fuzzy two sided ideal and g be any $(\epsilon, \epsilon \vee q_k)$ -fuzzy left ideal of S , then by Theorem 4, S is regular, so there exists $x \in S$ such that $a = axa$. Then we have:

$$\begin{aligned} (f \circ_k h \circ_k g)(a) &= (f \circ h \circ g)(a) \wedge \frac{1-k}{2} \leq (S \circ S \circ g)(a) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \{(S \circ S)(b) \wedge g(c)\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{S(p) \wedge S(q)\} \wedge g(c) \right\} \right) \wedge \frac{1-k}{2} \\ &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} 1 \wedge g(c) \right\} \right) \wedge \frac{1-k}{2} \\ &= \bigvee_{a=pqc} \left(g(c) \wedge \frac{1-k}{2} \right) \\ &\leq \bigvee_{a=pqc} \left(g(pqc) \wedge \frac{1-k}{2} \right) \\ &= g(a) \wedge \frac{1-k}{2} = g_k(a). \end{aligned}$$

Therefore, $(f \circ_k h \circ_k g)(a) \leq g_k(a)$. Also by Theorem

5, every $(\epsilon, \epsilon \vee q_k)$ -fuzzy generalized bi-ideal f of S is $(\epsilon, \epsilon \vee q_k)$ -fuzzy right ideal of S . Then we have:

$$\begin{aligned}
 (f \circ_k h \circ_k g)(a) &= (f \circ h \circ g)(a) \wedge \frac{1-k}{2} \leq (f \circ S \circ S)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{f(b) \wedge (S \circ S)(c)\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ f(b) \wedge \left(\bigvee_{c=pq} \{S(p) \wedge S(q)\} \right) \right\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{c=pq} \{f(b) \wedge (1)\} \right\} \right) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=bpq} \left\{ f(b) \wedge \frac{1-k}{2} \right\} \\
 &\leq \bigvee_{a=bpq} \left\{ f(bpq) \wedge \frac{1-k}{2} \right\} \\
 &= f(a) \wedge \frac{1-k}{2} = f_k(a).
 \end{aligned}$$

Therefore, $(f \circ_k S \circ_k g)(a) \leq f_k(a)$. And:

$$\begin{aligned}
 (f \circ_k h \circ_k g)(a) &= (f \circ h \circ g)(a) \wedge \frac{1-k}{2} \leq (S \circ h \circ S)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{(S \circ h)(b) \wedge S(c)\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} \{S(p) \circ h(q)\} \wedge S(c) \right\} \right) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \left\{ \bigvee_{b=pq} h(q) \wedge 1 \right\} \right) \wedge \frac{1-k}{2} \\
 &= \bigvee_{a=pqc} \left(h(q) \wedge \frac{1-k}{2} \right) \\
 &\leq \bigvee_{a=pqc} \left(h(pqc) \wedge \frac{1-k}{2} \right) \\
 &= h(a) \wedge \frac{1-k}{2}.
 \end{aligned}$$

Thus, $(f \circ_k h \circ_k g)(a) \leq h_k(a)$. Therefore, we have:

$$(f \circ_k h \circ_k g)(a) \leq f_k(a) \wedge h_k(a) \wedge g_k(a) = (f \wedge_k h \wedge_k g)(a).$$

Now, let $a \in S$. Then by Theorem 4, S is regular, so there exist elements $x, y \in S$, such that $a = axa$ and $ax = ya$. Thus we have:

$$ax = axaxax = axax(ya) = (axa)(xya).$$

Now:

$$\begin{aligned}
 (f \circ_k h \circ_k g)(a) &= (f \circ h \circ g)(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{a=bc} \{(f \circ h)(b) \wedge g(c)\} \right) \wedge \frac{1-k}{2} \\
 &\geq (f \circ h)(ax) \wedge g(a) \wedge \frac{1-k}{2} \\
 &= \left(\bigvee_{ax=pq} \{f(p) \wedge h(q)\} \right) \wedge g(a) \wedge \frac{1-k}{2} \\
 &\geq f(axa) \wedge h(xya) \wedge g(a) \wedge \frac{1-k}{2} \\
 &\geq f(a) \wedge h(a) \wedge g(a) \wedge \frac{1-k}{2} = (f \wedge_k h \wedge_k g)(a).
 \end{aligned}$$

Therefore, $(f \circ_k h \circ_k g)(a) \geq (f \wedge_k h \wedge_k g)(a)$. Hence $(f \circ_k h \circ_k g)(a) = (f \wedge_k h \wedge_k g)(a)$. It is clear that:

$$(iv) \Rightarrow (iii) \Rightarrow (ii).$$

Assume that (ii) holds. Let f be any $(\in, \in \vee q_k)$ -fuzzy quasi-ideal and g be any $(\in, \in \vee q_k)$ -fuzzy left ideal of S . Then, since S itself is an $(\in, \in \vee q_k)$ -fuzzy two sided ideal of S , by assumption we have:

$$\begin{aligned}
 (f \wedge_k g)(a) &= (f \wedge g)(a) \wedge \frac{1-k}{2} = (f \wedge S \wedge g)(a) \wedge \frac{1-k}{2} \\
 &= (f \circ S \circ g)(a) \wedge \frac{1-k}{2} = (f \circ (S \circ g))(a) \wedge \frac{1-k}{2} \\
 &\leq (f \circ g)(a) \wedge \frac{1-k}{2} = (f \circ_k g)(a).
 \end{aligned}$$

Then, from Theorem 6, S is regular. The rest of proof is same as in Theorem 10.

Conclusion

In this paper, we have characterized semi groups which are semi lattices of left groups using the properties of their $(\in, \in \vee q_k)$ -fuzzy two sided, interior, and quasi and bi-ideals. In our future work, we will concentrate on the characterizations of semi groups using the properties of their $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideals.

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Full Length Research Paper

Synthesis of perovskite CaTiO_3 nanopowders with different morphologies by mechanical alloying without heat treatment

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Mechanical alloying (MA) method is one of the methods used for large scale production of different nanopowders. In this study, calcium titanate (CaTiO_3 : CTO) nanoparticles have been synthesized via mechanical alloying (MA) without using heat treatment. The milled powders and CTO were characterized by XRD, SEM, and zetasizer. It is found that the CTO has a diameter of 30 - 70 nm with different morphologies. The results showed the minimum time of calcium titanate synthesis via mechanical alloying without heat treatment is 70 h that formed and the range of grain size (apparent size) using Williamson-Hall equation is 69 nm.

Key words: Mechanical alloying/activation, morphology, perovskite, CaTiO_3 .

INTRODUCTION

Calcium titanate (CaTiO_3 : CTO) belongs to the important group of compounds with a perovskite structure. Its most important features are high dielectric constant, large positive temperature of the resonance frequency, but also high dielectric loss that could be decreased by substitution of the A-site with trivalent ions (Evans et al., 2003). It is promising material for microwave tunable devices and is also used for modification of ferroelectric perovskites, such as PbTiO_3 or BaTiO_3 , for various applications (Kim, 2000; Ganesh and Goo, 1997). Calcium titanate is mostly prepared by a solid state reaction between CaCO_3 or CaO and TiO_2 at 1350°C , but also by some other methods such as sol-gel processing, thermal decomposition of peroxy-salts, and mechanochemical synthesis from different precursors, such as CaCO_3 , $\text{Ca}(\text{OH})_2$ or CaO , with TiO_2 (Vukotic et al., 2004; Mi et al., 1998). Up until now, various methods have been reported in the literatures for the syntheses of CaTiO_3 . These methods included: (a) conventional solid state

reaction between TiO_2 and CaCO_3 or CaO at a high temperature (Redfern, 1996; Chen et al., 2009), (b) mechanochemical methods (Mi et al., 2009; Brankovic et al., 2007; Palaniandy and Jamil, 2009), (c) chemical coprecipitation method (Gopalakrishna et al., 1975), (d) hydrothermal method (Wang et al., 2007; Li et al., 2009), (e) sol-gel route (Holliday and Stanishevsky, 2004; Zhang et al., 2008), and (f) polymeric precursor method (Pan et al., 2003). Among these methods, mechanical alloying (MA) is a solid-state powder process at ambient temperature and has been applied to synthesize different kinds of materials, such as crystalline, nanocrystalline, quasicrystalline and amorphous materials (Zoz, 1995; Suryanarayana, 2001).

Mechanical alloying, high-energy ball milling, has been used for many years now in producing ultra fine powders in the range of a sub-micron to a nanometer. Aside from size reduction, this process causes severe and intense mechanical action on the solid surfaces, which was

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known to lead to physical and chemical changes in the near surface region where the solids come into contact under mechanical forces (Venkataraman and Narayanan, 1998). These mechanically initiated chemical and physicochemical effects in solids were generally termed as the mechanochemical effect. In this work, Calcium titanate (CTO) is mostly prepared by a mechanical alloying between CaCO_3 and TiO_2 without heat treatment. The mechanical synthesis process is carried out in high intensity grinding mills such as vibro mills, planetary mills, and oscillating mills. It has been noticed that the size reduction process and the microstructural evolution of the CaTiO_3 during milling process were mainly influenced by the type of impulsive stress applied by the grinding media, which can either be an impact or shear type. Moreover, other parameters such as milling time, mill rotational speed and ball to powder at 3 different ratios affect the mechanical process. In fact, when the mechanical synthesis of the CaCO_3 and TiO_2 was carried out in planetary mills at higher ball to powder (70 : 1) ratio to produce CaTiO_3 , the impact stress was dominant, and not much attention was given on the mechanochemical mechanism itself. The aim of this work, therefore, is to give additional contribution in understanding the influence of milling conditions on the mechanical synthesis of CaTiO_3 nanoparticles without any the deleterious phase and heat treatment.

EXPERIMENTAL

Oxide powders of TiO_2 (99% < 1 μm , 99% purity) and CaCO_3 (99% < 1 μm , 99% purity) were used as raw materials which were mechanically ground in a purified air atmosphere. The ball-to-powder weight ratio was used at different ratios (20 : 1, 30 : 1 and 70 : 1). Mechanical alloying (MA) was carried out at ambient temperature and at a rotational speed (cup speed) of 350 rpm in a planetary ball mill. The mechanical alloying process was interrupted at regular intervals with a small amount of the MAed powder taken out from the vial to study changes in the microstructures at selected milling duration. The crystal phase was determined with powder X-ray diffraction. For these experiments, a Siemens diffractometer (30 kV and 25 mA) with the $K_{\alpha 1}$ radiation of copper ($\lambda = 1.5406 \text{ \AA}$), was used. The structural and compositional information of the product materials was obtained with scanning electron microscopy (SEM). The crystalline size (D) and lattice strain were estimated by Williamson-Hall (Williamson and Hall, 1953):

$$\beta \cos \theta = 2\varepsilon \sin \theta + 0.9 \frac{\lambda}{D}$$

Where λ is the wavelength of the X-ray, β the full width at half-maximum (FWHM), θ the Bragg angle, and ε is the microstrain. Finally, the particle size distribution of the powders was measured by zetasizer instrument (Malvern Co, HS C1330-3000, England).

RESULTS AND DISCUSSION

The XRD patterns of the samples consisting of TiO_2 and

CaCO_3 that had been ball milled for 0, 15, 20, 25, 40, 50, 60 and 70 h are illustrated in Figure 1. In the time of zero, only the TiO_2 and CaCO_3 peaks are observed. As shown in Figure 1, at 25 h, nothing significant takes place and only starting materials peaks are observed but in 40 h, all the peaks disappear because the material has become amorphous. In 40 and 50 h, we see the same situation. Because of the decrease of the particle size in milling the diffusion paths are shortened. Additionally, high energy is stored in the particles due to the cold work. Thus, the amorphous phase begins to grow around the crystals until all the material become amorphous. Due to the above, it seems that the mechanism of changing crystalline to amorphous in MA is diffusion controlled. There are reports showing that in some cases after amorphization, the crystalline phase has engendered again. As a result of the fact, that with the increase of milling time the kinetic energy of the systems intensifies, hence, the temperature increases which provides the needed energy for the reappearance of the stable state, i.e. crystallization (Koch, 1991). While rising the milling time, after 15 h, TiO_2 peaks disappear and CaTiO_3 peaks emerge. In this situation, the only distinguishable phase is CaTiO_3 . It seems that like an SHS reaction that needs a critical amount of energy to start and perform, in this case also, all TiO_2 and CaCO_3 have been transformed into CaTiO_3 due to the energy gained from milling. The thermal analysis of the 70 h milled sample showed no TiO_2 or CaCO_3 in the final composition of the synthesized powder to participate in reaction and therefore it seems that all reactants have changed to CaTiO_3 . Using the XRD patterns, the grain sizes were calculated. Figure 2 shows Williamson–Hall diagram of the system for 70 h and the mean size of the grains and the strain percentages are shown in Figure 2. In Figure 2, y represents $\beta \cos \theta$ and x represents $2 \sin \theta$ in Williamson–Hall equation. Hence, a as the slope represents the strain (η) and b as the y -intercept identifies $0.9\lambda/d$ from which the grain sizes (d) can be calculated. The grain size of CaTiO_3 were 69 nm for the milling time of 70 h. It is predicted that if the milling process continues, the grains become finer until they reach a critical value for the reason that MA process is the result of the competition of cold fusion and breaking of the components that causes the fineness and activation of the particles (Wang et al., 2001). At the critical point, the speeds of fusion and breaking balance out and the particles will no longer be fined (Ko et al., 2002).

According to SEM micrographs of the powders mechanically milled for 70 h in air atmosphere are shown in Figure 3a to h, that MAed powders are an ultra-agglomeration powder with approximately 100 ± 20 nanometers in size. Because, highly chemically active particles, these are strongly agglomerated. Interestingly, Figure 4a to d show that the product obtained after heat-treatment at 2 different temperatures (500 and 600°C) for 1 h is are mainly uniform special structures with suitable

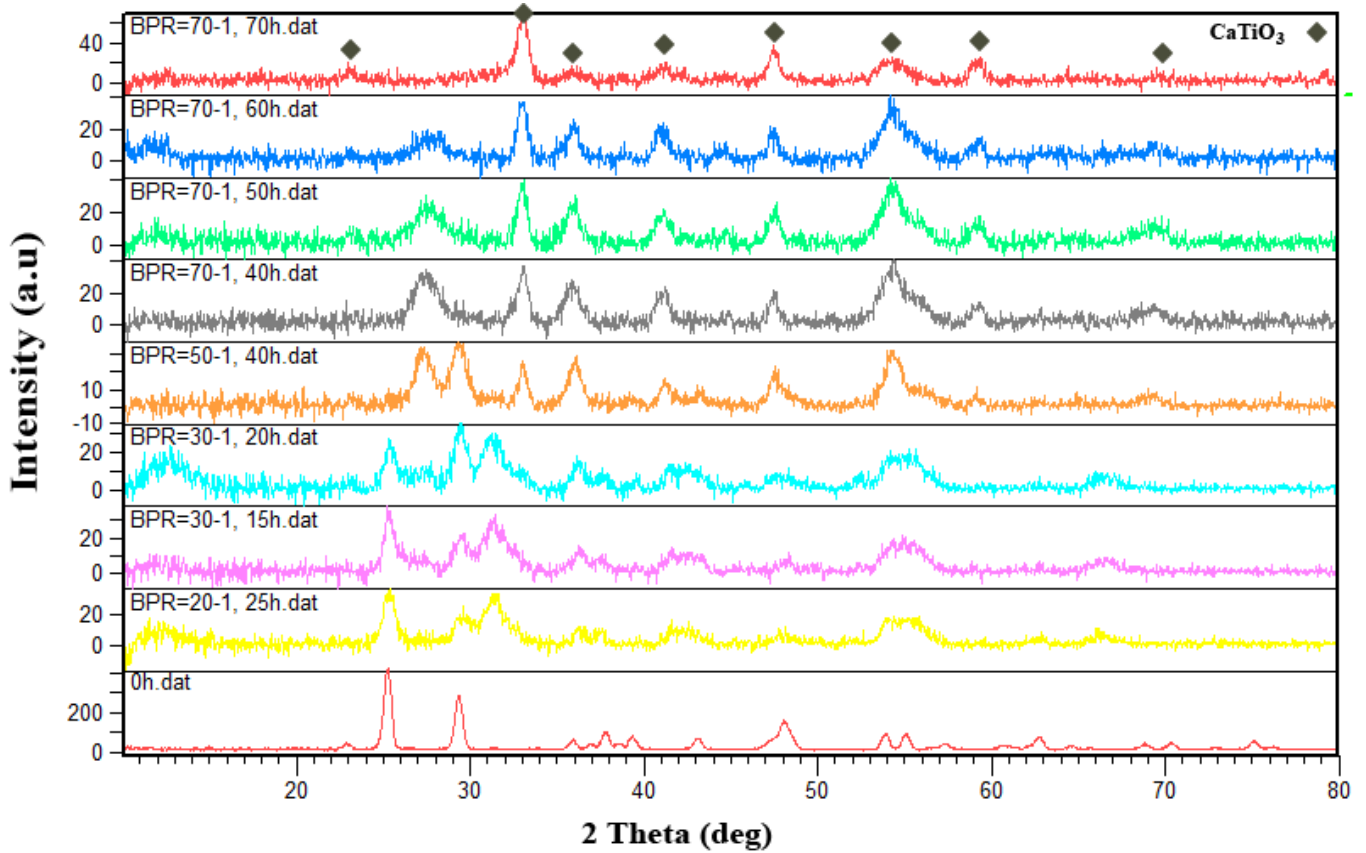


Figure 1. The X-ray diffraction spectra of mechanically alloyed CaCO₃/TiO₂ powders at different milling times.

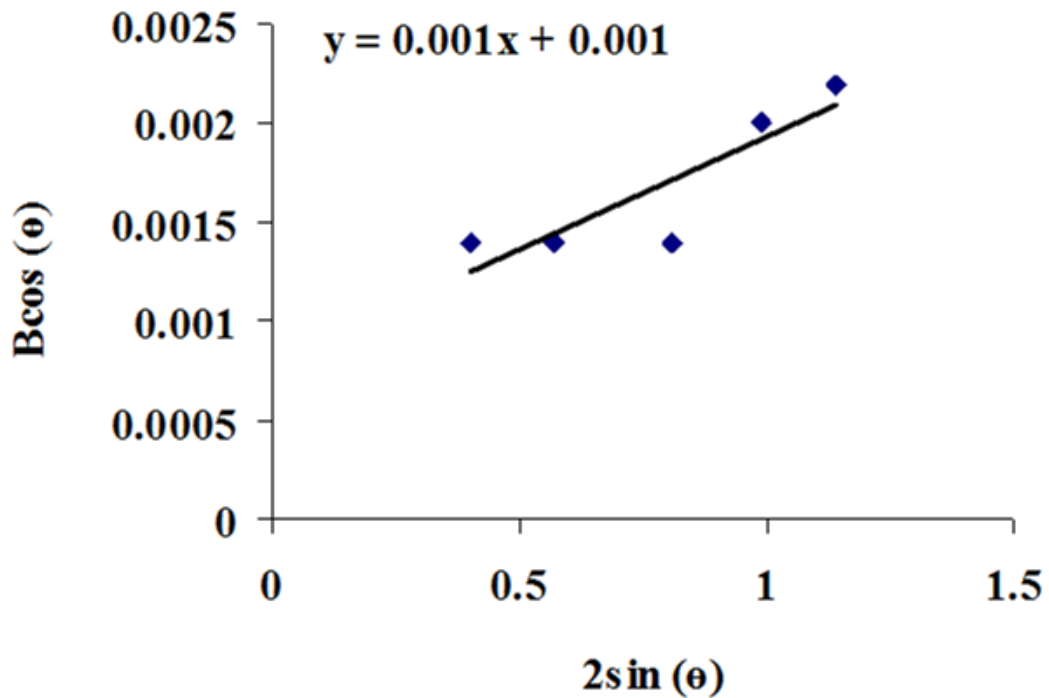


Figure 2. Calculation of strain and particle size in accordance to Williamson–Hall equation for CTO after 70 h of ball milling.

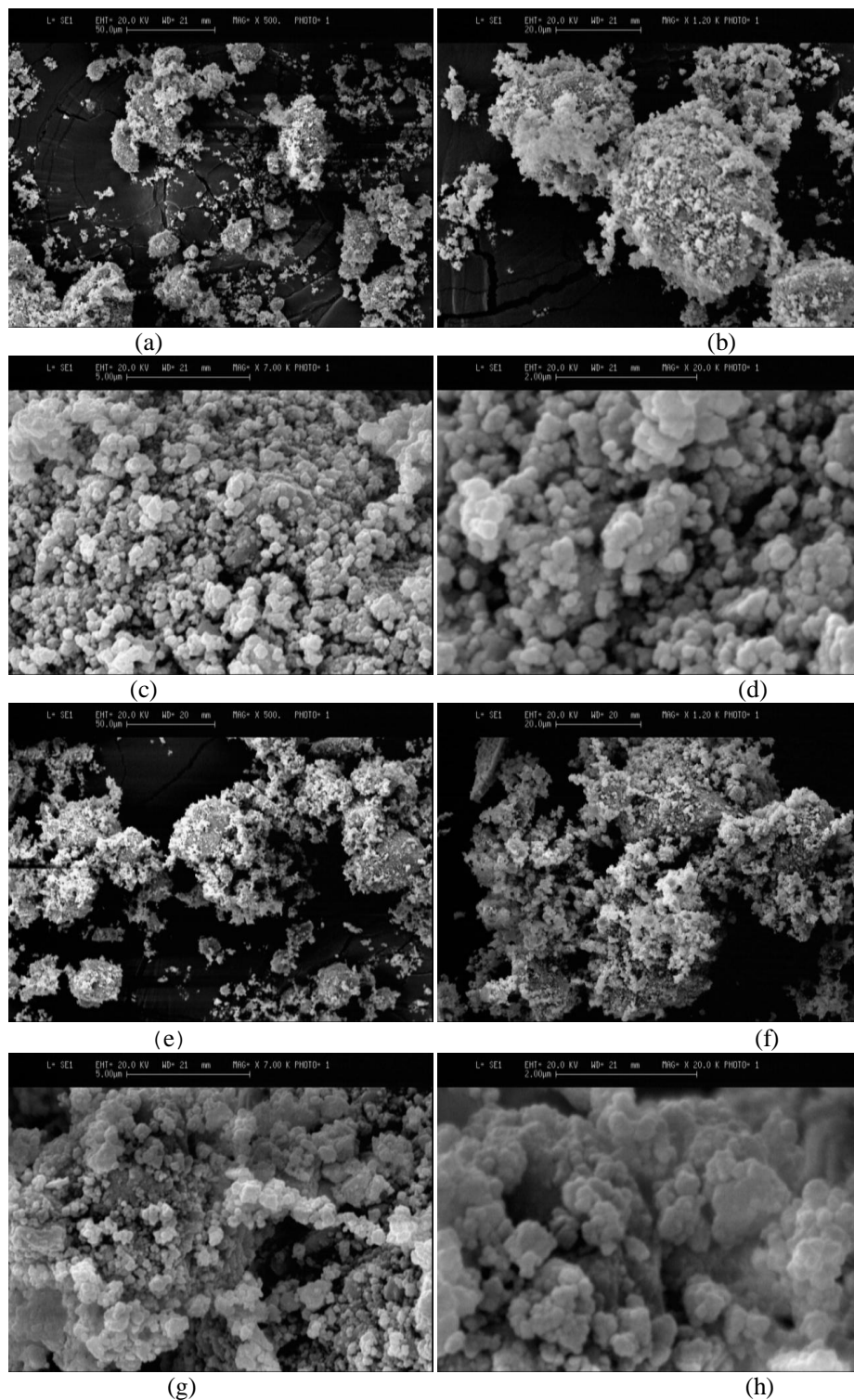


Figure 3. SEM images of milled samples in 70 h at different magnifications.

crystallinity grades with a diameter of 60 to 90 nm, which is of very extraordinary uniform morphologies. Finally, in this investigation, an effective method was developed for the formation of ultra-crystallinity with uniform

morphologies. As the matter of fact, this method (MA) guarantees its production in the synthesis of CTO for different applications.

The nanoparticle size of CTO milled (70 h) product was

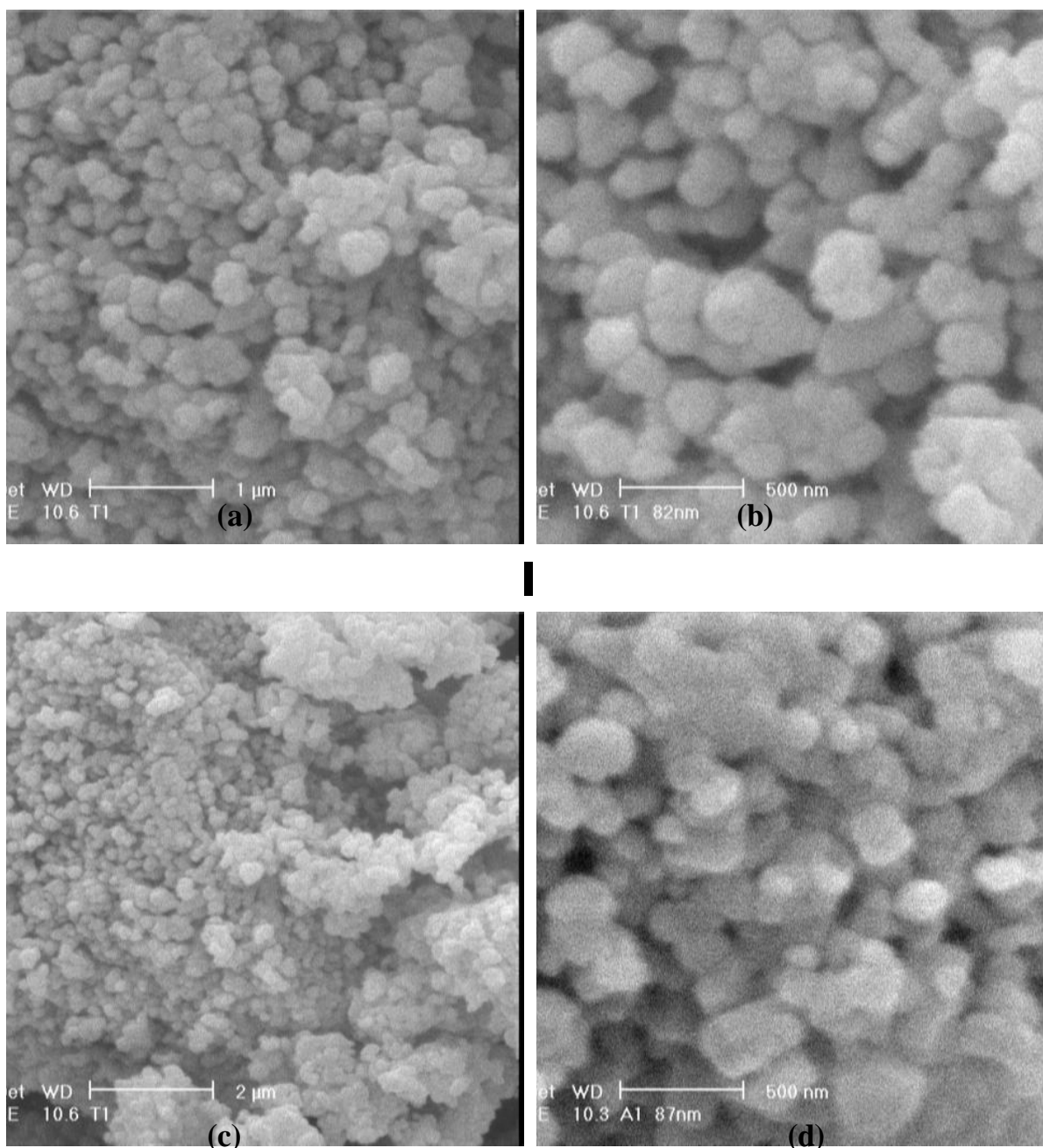


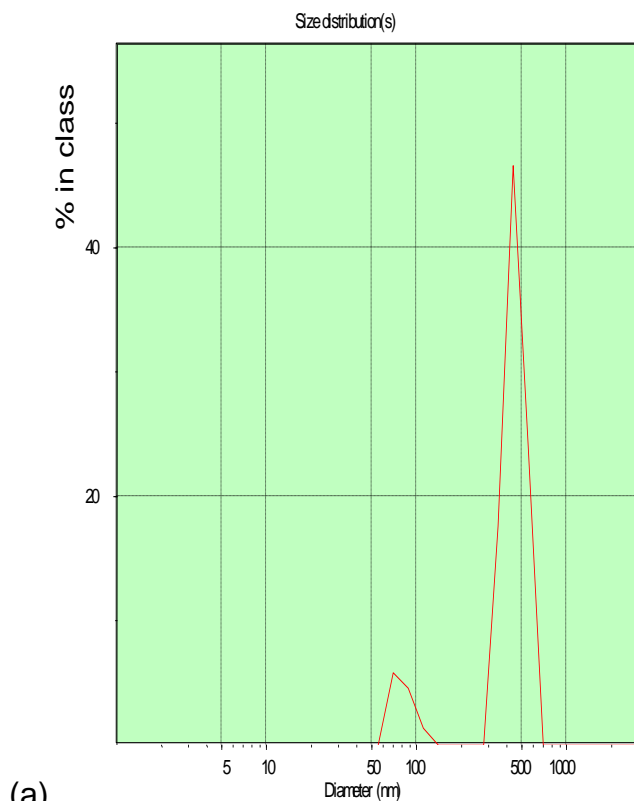
Figure 4. SEM different images after heat-treatment at different annealing temperatures with different magnifications a-b) 500°C, c-d) 600°C.

analyzed using a zetasizer method. These measurements reveal the particles to be highly wide distribution (Figure 5a). The milled CTO powders were particles with diameters 2 ranging from 55 to 100 nm and 300 to 550 nm. Figure 5b shows the zetasizer curves of the CTO powders obtained from the heat treatment for 2 h and 500°C. As can be observed in these images, the particle sizes grow up with increasing the aging time. The

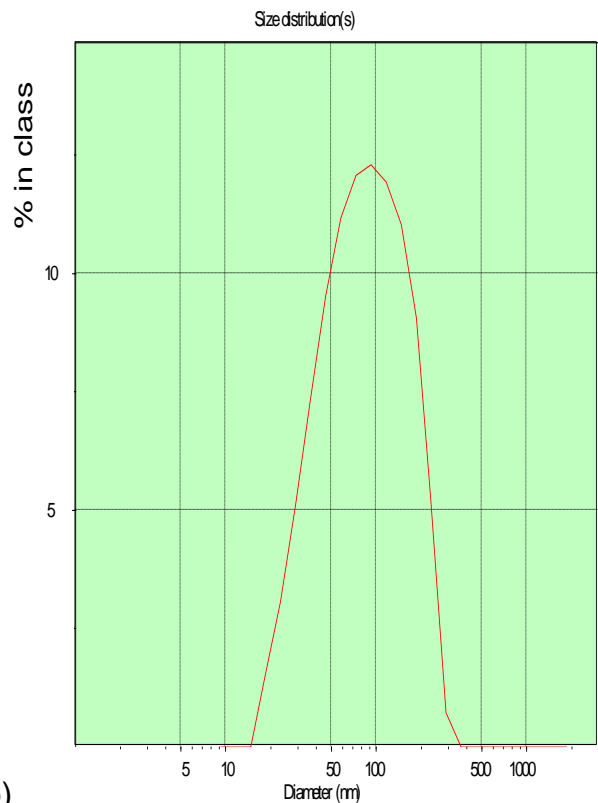
average particle sizes of powders aged for 2 were regular and uniform.

Conclusion

CTO with different morphologies was synthesized by a MA method. The purity and good quality of CTO obtained



(a)



(b)

Figure 5a and b. Zetasizer images of the CTO powders obtained from the MA a) only 70 h of ball milling, b) 70 h of ball milling with heat treatment in 500°C.

by MA make it a promising method for the production of CTO. The synthesis of CTO was strongly dependant on the experimental parameters such as milling time and ball to powder ratio. Optimal conditions of CTO synthesis were selected as 70 : 1 ratio and 70 h of milling time. This simple approach should promise us a future large-scale synthesis of this nanostructured materials for many important applications in nanotechnology in a controlled manner.

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Full Length Research Paper

Hurricanes and cyclones kinematics and thermodynamics based on Clausius-Clapeyron relation derived in 1832

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Juxtaposing Clausius-Clapeyron relation derived in 1832 with Hydrodynamic concept of atmosphere parcel of air leads to the discovery of an essential property of the troposphere that will deeply ameliorates information contained in literature and audiovisual productions on tropical weather. It is indeed a rectification of the use of an ideal gas principle which enshrined the idea that, hot air is lighter than cold air throughout the atmosphere. In other words, contrary to what has been taught in schools and universities of the world, hot air is not lighter than cold air in all parts of the troposphere. Taking into account this troposphere thermodynamics reality improves our understanding of complex weather phenomena such as cyclones and hurricanes. The two equal level surfaces of water vapor and temperature rating respectively at 6.11 mb and 0.0098°C separate without any ambiguity parts of the troposphere where ideal gas assumption can be applied to parts of the troposphere where this assumption is banned.

Key words: Clausius-Clapeyron relation, Hydrodynamic concept of air parcel, equal level surfaces, rating at 6.11 mb and 0.0098°C.

INTRODUCTION

A number of questions regarding Kinematics and thermodynamics of hurricanes and cyclones remain unanswered despite the quality and quantity of ground-or space-based observations. Instead these weather phenomena are combinations of complex troposphere physical processes that occur under the accuracy of temperature and humidity conditions. In this study, regardless of the manner in which hurricanes and cyclones are consider (Arakawa and Suda, 1953; Ballenzweig, 1957; Bangs, 1929; Beerbower, 1926; Cline, 1926; Conner et al., 1957; Duane, 1935; Dunn, 1956;

Fassig, 1913; Fletcher, 1955; Gentry, 1955; Haurwitz, 1935; Hoover, 1957; Hughes, 1952; Jordan, 1952; Klein and Winston, 1947; Malkin and Galwaym, 1953; Malkus, 1958; McDonald, 1942; Miller, 1958; Riehl and Palmen, 1957; Rossby, 1949; Schoner and Molansk, 1956; Tannehill, 1936), we want to make a contribution to a better understanding of kinematics, and thermodynamics governing these weather events with high destructive power. Our results are obtained from effectiveness of Clausius-Clapeyron equation that leads to the slope of the equilibrium curves in the pT -plane (Figure 1) whose

show precisely that, unlike the dry water vapor that can be assimilated to the ideal gas at all times, saturated water vapor at low temperatures (temperature below 0.0098°C) in the presence of high humidity of air (vapor pressure above 6.11 mb), has thermoelastic properties diametrically opposed to those of ideal gases (including dry water vapor). In tropical regions, saturated water vapor occupies the middle and top of the troposphere to more than 90% (it should be noted that, saturated water vapor is known as the birth place, home or bed of weather events such as hurricanes and cyclones or clouds related). Therefore, it was necessary to take account of this characteristic property of the saturated water thermodynamics to successfully draw new and unique profiles of hurricanes and cyclones.

Names assigned to tropical disturbances and related precipitating systems vary from one community to another. “*Tornado*” in French is used to refer to *cold-disturbances* while “*Tornado*” in Anglo-Saxon community is used to refer to *hot-disturbances*. In the translation of literature and audiovisual productions, this distinction is not often made and leads to inconsistencies and confusion. Indeed, vertical profiles of cold disturbances are diametrically opposite directions vertical profiles of hot disturbances. Hurricanes or tornadoes will be referred to hot-disturbances while cyclones will be referred to cold-disturbances in this work.

TROPOSPHERE DYNAMIC BALANCE

Atmosphere dynamics uses precise concept of air particle (Batchelor, 1967; Riegel, 1992). Especially:

- Few exchanges on molecular scale: it is easy to follow quantity of air which preserves certain properties.
- Quasi-static equilibrium: at any moment there is dynamic balance, that is, the particle has the same pressure as its environment ($P = P_{ext}$).
- No thermal balance: heat transfers by conduction are very slow and neglected. One can have $T \neq T_{ext}$.
- The horizontal sizes of the air particle can go from a few cm to 100 km according to the applications.

Taking into account the fact that, the atmosphere is mainly composed of dry air and water vapor, the Dalton’s law connects the pressure (P) with the partial pressure of dry air (P_a) and water vapor (e):

$$P = P_a + e \quad (1)$$

In deriving P with respect to the temperature, one has

$$\frac{dP}{dT} = \left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial P}{\partial V}\right)_T \cdot \left(\frac{dV}{dT}\right) \quad (2)$$

According to the quasi-static equilibrium (or dynamic balance), the pressure in the air particle must be the

same as that of the air around, including during its water contains changes in phases. In other words, P in the air particle remains constant during individual changes in phases. Hence:

$$\frac{dP}{dT} = 0 \quad (3)$$

Equations 2 and 3 lead to the derivative of V compared to T:

$$\frac{dV}{dT} = -\frac{\left(\frac{\partial P}{\partial T}\right)_V}{\left(\frac{\partial P}{\partial V}\right)_T} \quad (4)$$

Introducing (χ) the coefficient of thermal expansion of moist air at constant temperature:

$$\chi = -\frac{1}{P} \left(\frac{\partial P}{\partial V}\right)_T \quad (5)$$

Then the equation of atmosphere dynamic balance:

$$\frac{dV}{dT} = \frac{1}{\chi} \cdot \frac{1}{P} \left(\frac{\partial P}{\partial T}\right)_V \quad (6)$$

Using Clausius-Clapeyron estimations of $\left(\frac{\partial P_a}{\partial T}\right)_V$ and $\left(\frac{\partial e_w}{\partial T}\right)_V$, Equation 6 of troposphere (or birth place of weather events) dynamic balance become:

$$\frac{dV}{dT} = \frac{1}{\chi} \cdot \frac{1}{P} \left(\frac{\partial e_w}{\partial T}\right)_V \quad (7)$$

Equations 6 and 7 lead to a very important atmosphere dynamics statement; at any moment and throughout the atmosphere, one can use Equations 6 or 7 and Clausius-Clapeyron slope of the equilibrium curves in the pT-plane (Figure1) to predict in which direction the air parcel will move (up or down) if its temperature increases or decreases. Table 1 and Figure 2 provide an overview of possible situations in the troposphere.

BASIC KINEMATICS AND THERMODYNAMICS OF HURRICANES AND CYCLONES

Vertical profiles of hurricanes

Hurricanes are triggered by passive deep convection generated by a hot source located at the surface of the

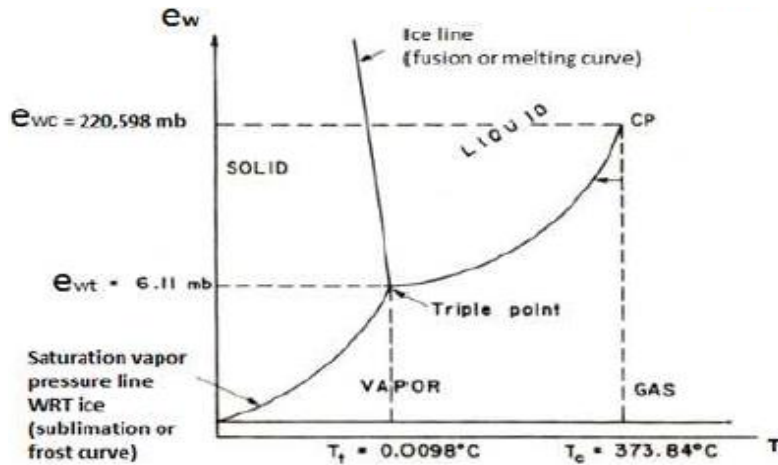


Figure 1. Saturation curves for water substance onto the p_T -plane. (p_T and T_T are called triple-point data): $e_{wT} = 6.11 \text{ mb}$; $T_T = 0.0098^\circ\text{C}$.

Table 1. Changes in volume of the moist air particle depending on temperature within a specific range of temperature and humidity.

Range of temperature coupled with range of humidity	$T < 0.0098^\circ\text{C}$ $e_w < 6.11 \text{ mb}$	$T < 0.0098^\circ\text{C}$ $e_w > 6.11 \text{ mb}$	$T > 0.0098^\circ\text{C}$ $e_w > 6.11 \text{ mb}$
$(\frac{\partial P}{\partial T})_v$	+	-	+
$\frac{dV}{dT}$	+	-	+

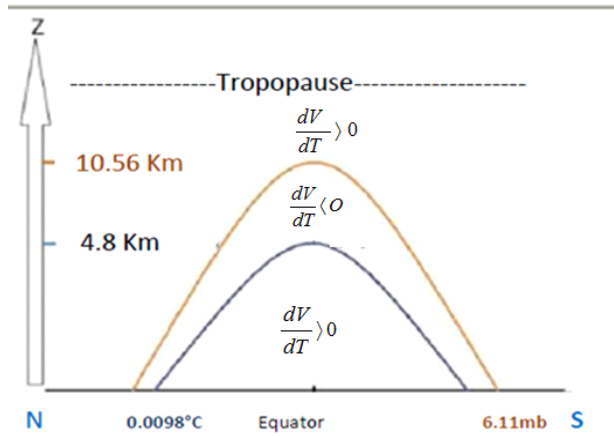


Figure 2. Troposphere specific regions depending on the manner in which V changes with T (V and T are respectively volume and temperature of an air parcel): $\frac{dV}{dT} > 0$; the particle swells when its temperature increases (so it becomes lighter). $\frac{dV}{dT} < 0$; the particle shrinks when its temperature increases (so it becomes less light). -10.56 Km = maximum elevation (statistic value) of 6.11 mb pseudo-isobar, -4.8 Km = maximum elevation (statistic value) of 0.0098°C isotherm, -V (air parcel volume), T (air parcel temperature).

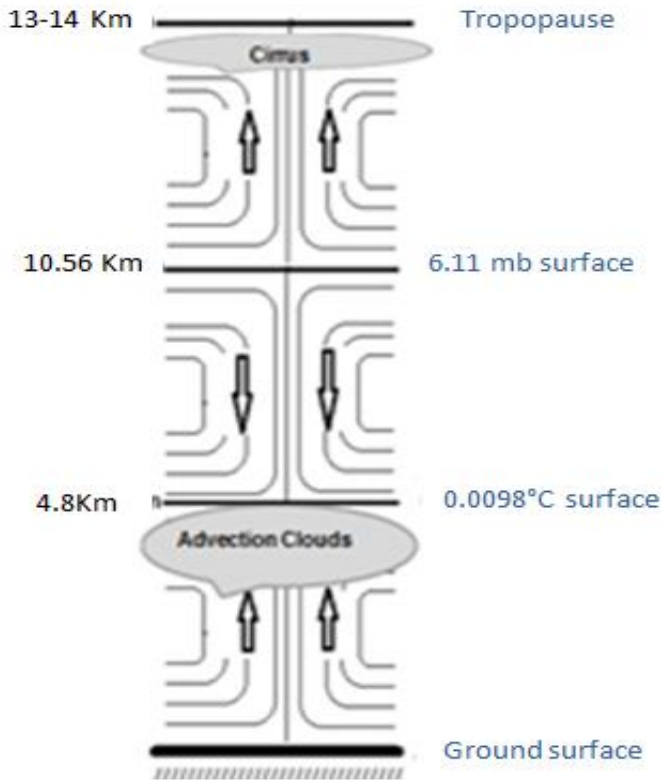


Figure 3. Hurricanes are triggered by passive deep convection generated by a heat hot source located at the surface of the Ground. They appear as high towers consisting of three floors: warm updrafts occupying floors 1 and 3 while warm downdrafts occupy floor 2.



Figure 4 (a-d). Hurricanes or tornadoes trigger dust clouds whose base is thin compared to the peak which is very broad. Hurricanes (or Tornadoes) can also electrify (Mbane, 2012) the troposphere column in which it is formed (Figure 4c). Its broadest peak indicates the presence of hot downdrafts that prevent the progression of warm updrafts.

earth and appear (Figure 3) as very high towers (from 0 to about 9 km) consisting of 3 floors: warm updrafts occupying the first and third while warm downdrafts occupies the second floor. According to ground based observations of Figure 4(a to d), over-land hurricanes (or tornadoes) trigger clouds whose base is thin compared to the peak which is very broad. Hurricanes vertical drafts can also produce electrical charges (Mbane, 2009) in the troposphere column in which it is formed (Figure 4c). The broadest peaks of the related clouds indicate the presence of the second floor warm downdrafts that prevent the progression of the first floor warm updrafts (Figure 3). Considering the molecular scale, our model (Figure 4e) based on Clausius-Clapeyron's relation (1832) suggests, unlike ideal gas cumulonimbus model (Figure 4f), blocking of hot updrafts by hot down drafts which means installation of an additional greenhouse effect, which causes the accumulation of cloud formation latent heat with earth's surface radiate heat R_T ($R_T = \epsilon_s \sigma T_s^4$) at the ground surface. This is consistent with based observations and explains high surface temperatures that accompany the formation of clouds in the sunny sky.

Vertical profiles of cyclones

Cyclones are triggered by very deep and passive convection generated by a cold source (squall-line) located at the summit of the troposphere and appear (Figure 5) as very high towers (from 0 to about 14 km) consisting of 3 floors: cooler downdrafts occupying 1st and 3rd while cooler updrafts occupy the 2nd floor. There is good agreement between aircraft-based observations and related cyclones second floor updrafts convective clouds whose base has to be located above 0.0098°C isotherm surface. Cyclones vertical drafts can also produce electrical charges (Mbane, 2012) in the troposphere column in which it is formed.

Horizontal profiles of hurricane and cyclone

Observed pressure near the eyes of cyclones (or hurricane) is very low and concentrates a rapid decrease in a short distance so that the momentum of particles of air, the frictional force and the tidal force are (from surface of the earth to tropopause) negligible compared to the coriolis and pressure-gradient forces. When pressure gradient and coriolis forces are the only two factors acting, geostrophic winds (rotative in the Northern hemisphere and contra-rotative in the Southern hemisphere) immediately take place (Figures 6) within deep and passive convections. The impact of hurricanes footprint (less than a dozen kilometers in diameter) is much lower than that of cyclones (several tens of kilometers in diameter).

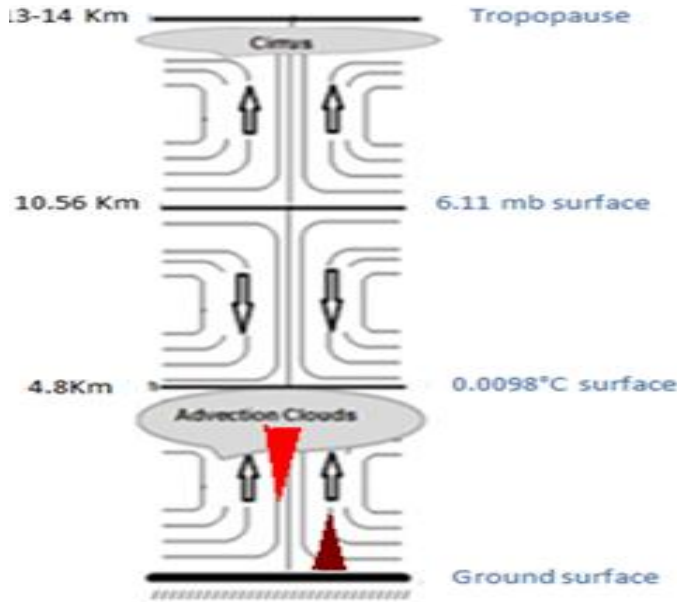


Figure 4e. Our model based on Clausius-Clapeyron's relation (1832) suggests blocking of hot updrafts by hot down drafts: then install an additional greenhouse which triggers the superposition of cloud formation latent heat (red color) with earth's surface radiate heat $\epsilon_s \sigma T^4$ (brown color). This is consistent with observations and explains higher temperature and humidity of atmosphere lower layers and tropical regions moderate climate.

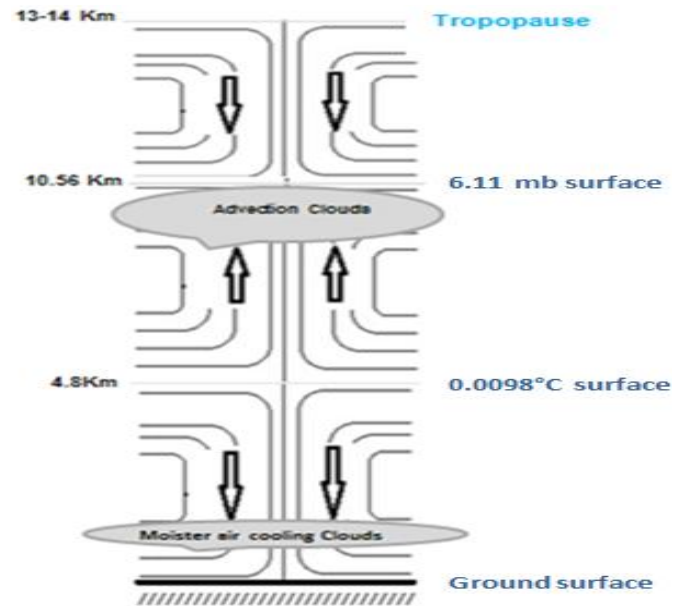


Figure 5. Cyclones are triggered by passive deep convection generated by a cold hot source located at the mid- troposphere. They appear as high towers consisting of three floors: Cold updrafts occupying floors 2 while cold downdrafts occupy floor 1 and 3

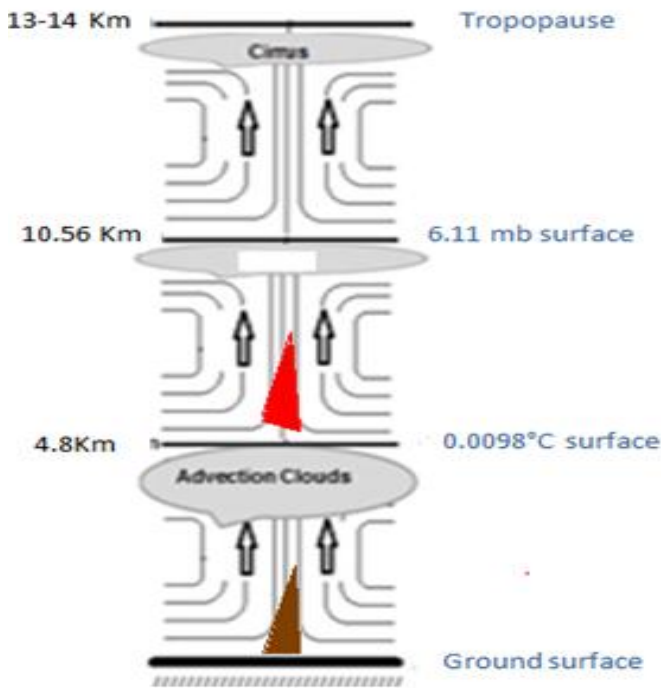


Figure 4f. Ideal gas properties suggest passive convection transfer from earth's surface to tropopause and automatically excluded the spread of cloud formation latent heat to that earth's surface. There is therefore no possibility of accumulation of heat or water vapor in lower troposphere. This is not consistent with earth's atmosphere physics.

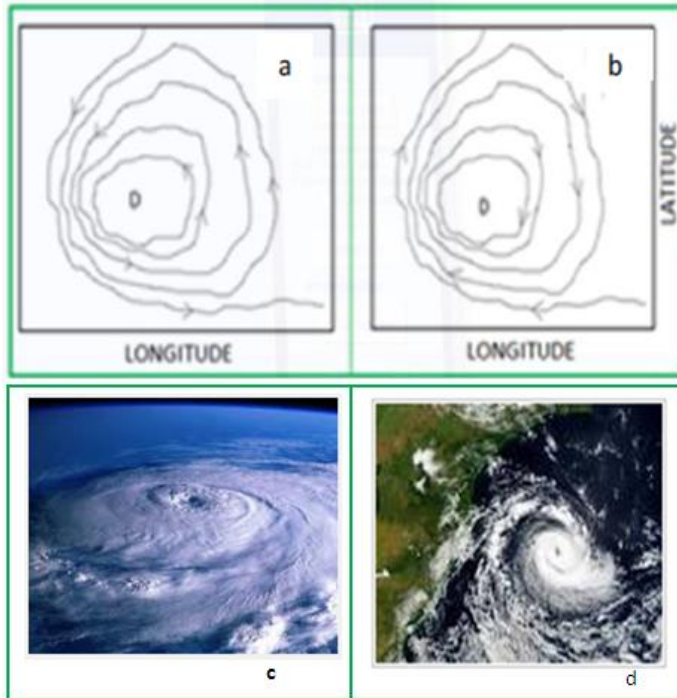
TROPOSPHERE GRAVITY WAVES AND LOCAL PRESSURE FLUCTUATIONS

In fluid medium, movement from point A to point B of local pressure fluctuations (δP) are provided by gravity waves. For incompressible fluids (including the atmosphere because of its elasticity), these waves travel at phenomenal speeds. In the troposphere, the propagation is essentially straight and parallel to the direction of gravity acceleration (\vec{g}). Therefore, when a surface depression (D) occurs, it is almost spontaneously covered by altitude low pressure (L).

Reciprocally, when L occurs, it is immediately relayed by a D. This is same for Anticyclone (A) and high pressure (H). In the troposphere, D and L appear without any indication of which the two came first. Hurricanes (or tornadoes) are caused by warming ground surface areas, while cyclones are caused by mid-troposphere cooling domains. Due to gravity waves, satellites in their current configuration cannot differentiate between hurricanes and cyclones. This is a very embarrassing situation that leads to numerous confusion and makes ineffective "Weather Alerts Systems".

Conclusions

The troposphere, generally known as birth place of weather phenomena, is a huge thermodynamic engine driven by the energy received from the sun. All winds,



Figures 6. Streamlines of geostrophic winds triggered by cyclones around their low pressure groove. Direct rotation is observed in the northern hemisphere (e.g. a and c) while indirect rotation is observed in the southern hemisphere (e.g. b and d): based on trigonometric considerations.

storms and clouds result from the differences in the amount and utilization of this energy. Since the radiant energy appears principally as heat, it was necessary to resort the thermodynamics properties of saturated water vapor in order to better understand how the moist air reacts to heat changes in any portion of the troposphere. New and unique kinematic profiles of hurricanes and cyclones can now be easily plotted. It should be noted that, all natural meteorological phenomena included hurricanes and cyclones can be traced to the manner in which the energy from the sun is received over different parts of the earth.

Since the troposphere is a medium in which mass motions are easily started, convection is found to be one of the chief ways in which heat is transferred there. This transfer may be accomplishing either by vertical or by horizontal motions. According to our results: warmer disturbances that occur in lower-troposphere are dissipated (that is, Atmosphere is a force-restore engine or dissipative system) by a typical mass motion usually called hurricanes (or tornadoes) while cooler disturbances that occur in mid-troposphere are dissipated by another typical mass motion called cyclones. Knowing that coriolis force act to west on updrafts, everyone can now understand why hurricane and cyclone move preferentially from East to West due to the localization in updrafts of their heat sources.

Cyclones' heat source is made of huge and cooler fogs (those observed temperatures are less than -45°C) which can travel even increase (over hot oceans) in the troposphere while hurricanes' heat source is fixed on the Ground: that's why cyclone lives and travels longer than hurricane. Moreover, contrary to a widespread idea in meteorology that warm air is less dense than cold air, this work shows that: between the two equal level surfaces of water vapor and temperature respective rating 6.11 mb and 0.0098°C , hot air is less light than cold air. This explains the presence of cold upwelling in cyclones and hot downdrafts in hurricanes. We cannot conclude this work without strongly highlighting the fact that hot air, contrary to what is taught until now, is not lighter than cold air in all parts of the troposphere. This troposphere's thermodynamic property is unfortunately unknown to public and has led to numerous errors and inconsistencies that abound in many scientific books and publications (including audiovisuals productions). In the next investigation on climate, using only certainties (e.g. ideal gas approximation imposes a partition of the troposphere into 3 regions, vertical temperature gradient in the troposphere has a negative sign, moist air condenses as it cools, pressure in the air parcel is the same everywhere and is equal to that of its immediate neighbors in all circumstances (that is, state of atmosphere quasi static equilibrium) instead of wrong presumptions (e.g. ideal gas approximation is valid throughout the atmosphere, warm air is lighter than cold air throughout the atmosphere, updrafts are necessarily associated with warming, downdrafts are necessarily associated with cooling) would allow:

- To deeply ameliorate our view of troposphere phenomena regardless of their spatial and temporal scales (e.g. Rossby's suggested representation of general circulation),
 - To greatly exorcise our fears (sometimes ridiculous) generally triggered by a lack of explanations devoid of ambiguity,
 - To better protect ourselves from disasters caused by these devastating events, those life cycles until now unfortunately escapes the human control.
- Knowing physics processes behind devastating event makes it less daunting (a good example is experienced in Mexico where building's architecture is gradually adapted to the local soil structure which paradoxically amplifies seismic waves which comes from far away, instead of reducing their intensity).

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Full Length Research Paper

A comparative study of biogas production using plantain/almond leaves and pig dung, and its applications

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Plantain/almond leaves and pig dung were used as substrates in anaerobic biodigester for producing biogas by batch operation method within the mesophilic temperature range of 20.0 to 31.0°C. The study was carried out to compare biogas production potential from plantain/almond leaves and pig dung wastes. The cumulative biogas produced from the plantain/almond leaves was 220.5 L while the cumulative biogas from the pig dung was 882.5 L. Orsat apparatus was used to analyze the gas produced. The methane component of gas from pig dung was 70.2% while that for plantain/almond leaves with algae was 72.7%. The biogas from the almond/plantain leaves became combustible on sixteenth day while the biogas from the pig dung was combustible on fourteenth day. Results showed that pig dung produced more biogas than the almond/plantain leaves within the same period.

Key words: Mesophilic, anaerobically, almond/plantain, combustible, algae.

INTRODUCTION

Biogas originates from the process of bio-degradation of organic material under anaerobic (without air) conditions. In the absence of oxygen, anaerobic bacteria decompose organic matter and produce a gas mainly composed of methane (60%) and carbon dioxide called biogas. This gas can be compared to natural gas, which is 99% methane. Biogas is a 'sour gas' in that it contains impurities which form acidic combustion products (Boyd, 2000). In 2008, about 19% of global final energy consumption came from traditional biomass, which is mainly used for heating and 3.2% from hydroelectricity (Ross, 1996). In other words, animal and agricultural wastes constitute a high proportion of biomass and their utilization is important for economical and environmental aspects possessing suitable climatic and ecological

conditions (Oktay, 2006).

Biogas as a renewable energy source could be a relative means of solving the problems of rising energy prices, waste treatment/management and creating sustainable development. Anaerobic digestion has been recognized as an effective way to partially solve the growing concern of solid waste management by reducing the weight of organic waste, as well as controlling the soil pathogens (Ishita and Sen, 2013). The production of biogas involves a complex biochemical reaction that takes place under anaerobic conditions in the presence of highly sensitive microbiological catalyst that are mainly bacteria. Biogas technology has in the recent times also been viewed as a very good source of sustainable waste treatment/management, as disposal of wastes has

become a major problem especially to the third world countries. The effluent of this process is a residue rich in essential inorganic elements needed for healthy plant growth known as bio-fertilizer which when applied to the soil enriches it with no detrimental effects on the environment (Energy Commission of Nigeria, 1998). The raw materials used in many places for the gas production include agricultural wastes and animal manures which are called Biomass. The greatest potential to increase the use of biomass in energy production seems to lie in forest residues and other biomass resources e.g. agro biomass and fruit biomass (Kramer, 2002; Eija et al., 2007). The plantain/almond leaves, algae from sewage and pig dung used in this research work are readily available in our environment. These biomasses are highly degradable in nature. However the rate of efficiency of digestion of feedstock depends on its physical and chemical form.

Plant materials especially crop residues are more difficult to digest than animal manures. This is because hydrolysis of cellulose materials of crop residues is a slow process and can be a major determining step in anaerobic digestion process. Raw plant materials are bound up in plant cells usually strengthened with cellulose and lignin, which are difficult to digest. In order to let the bacteria reach the more digestible foods, the plant material must be broken down (Kozo et al., 1996; Fulford, 1998). Addition of bacterial seed or inoculum accelerates biogas generation and also reduced the lag time (number of days required for biogas production to start). The inoculum is known to enrich the bacterial of the digester which will enhance their action on the substrate and hence on the quantity as well as quality of the biogas generated. Biogas microbes play different roles in the conversion of the organic substances, according to their nutrient requirements (Maishanu and Maishanu, 1998). Maximum methane yield requires adequate and efficient nutrient supply for microorganisms in the digester (Thomas et al., 2006).

Plantain leaves (*Musa paradisiaca*) are readily available in the tropics, while the almond tree (*Terminalia catappa*) popularly known as fruit or umbrella tree are found around the school premises (University of Nigeria, Nsukka), their wastes constitutes a nuisance in the school environment. However, these wastes can be converted to a renewable energy source. This experiment was carried out to find out the effect of parameters such as pH level, temperature, retention time and the biodegradability of these biomass. When the temperature is high, the activity of bacteria is simply more vigorous, so that the fermentation period becomes shorter. When the temperature is low digestion is slow, and the fermentation period is longer (Goodger, 1980).

The anaerobic digestion processes is carried out by a delicately balanced population of various bacteria. These bacteria can be very sensitive to change in their environment. Temperature is a prime example. It has been determined that 35°C is an ideal temperature for

anaerobic digestion (Kucha and Itodo, 1998).

AIMS AND OBJECTIVES

This paper determined the effect of environmental and operational parameters on the fermentation/production rate of biogas from organic waste. It also studied the extent to which plantain leaves/almond leaves; algae from sewage pond and pig dung generate gas, and transformed the organic wastes into high quality fertilizer.

The chemistry of biogas production

Generally, the production of this biogas involves a complex biochemical reaction that takes place under anaerobic conditions in the presence of highly sensitive microbiological catalysts that are mainly bacteria. The major products of this reaction are methane (CH₄) and carbon-dioxide (CO₂) (Hashimoto et al., 1980). The anaerobic biological conversion of organic matter occurs in 3 stages (Figure 1).

Factors affecting biogas production

Many factors that are affecting the fermentation process of organic substances under anaerobic condition include:

- i. Temperature
- ii. Nature of raw material
- iii. pH of slurry and alkalinity
- iv. Stirring
- v. Carbon/nitrogen ratio (C/N)
- vi. Nutrients addition
- vii. Retention time
- viii. Total solid
- ix. Volatile solid
- x. Mixing
- xi. Inhibition.

The length of fermentation period is dependent on temperature. Keeping the digestion chamber at nearly constant temperature is important.

Smaller particles would provide large surface area for adsorbing the substrate that would result in increased microbial activity and hence increased gas production. pH is an important parameter affecting the growth of microbes during anaerobic fermentation. Stirring of digester content needs to be done to ensure intimate contact between microorganisms and substrate which ultimately improves digestion process. It is generally found that during anaerobic digestion microorganisms utilize carbon 25 to 30 times faster than nitrogen. Thus to meet this requirement, microbes need a 20 – 30: 1 ratio of C to N with the largest percentage of the carbon being readily degradable (Bardiya, and Gaur, 1997; Malik and Tauro, 1995). Addition of inoculum tends to improve both

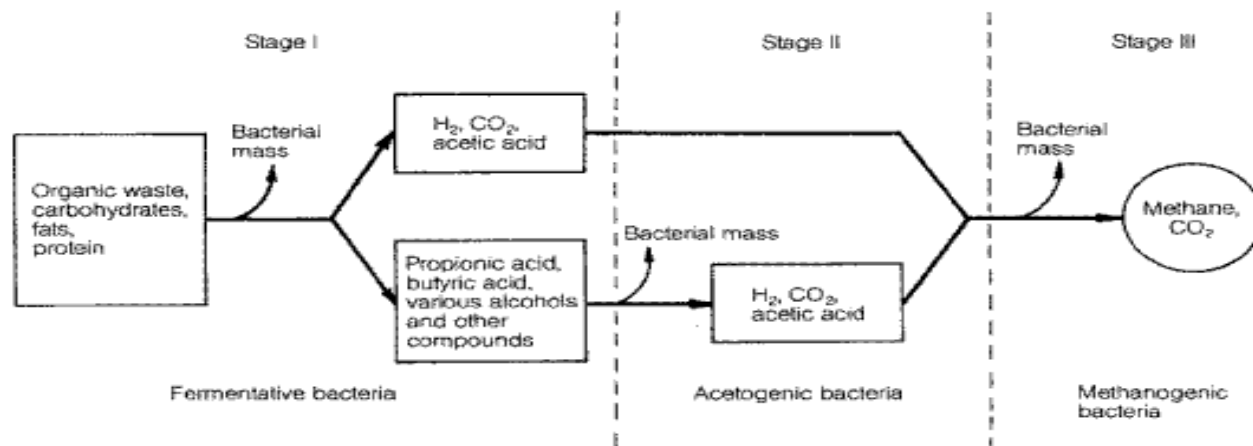


Figure 1. The three-stage anaerobic fermentation of biomass (Hoerz et al., 2008).



Figure 2. Digester used in the experiment.

the gas yield and methane content in biogas. It is possible to increase gas yield and reduce retention period by addition of inoculums (Dangaggo et al., 1996; Kanwar and Guleri, 1995; Kotsyurbenko et al., 1993). Retention time is the average time spent by the input slurry inside the digester before it comes out. Total solid concentration (TS%) is a measure of the dilution ratio of the input material. Some organic materials which contain lignins do not decompose easily. The lignin content even if quite low will decrease the rate of digestion of carbohydrates (cellulose and hemicellulose).

MATERIAL AND METHODS

Apparatus

The biogas digester that was used for this study is 0.00147 m³ in capacity and made of galvanized metal sheet material (Figure 2).

Batch operation method was used. Pre-decayed plantain/almond leaves and pig dung were used for this study. The plantain/almond leaves were obtained from University of Nigeria, Nsukka premises. Pig dung was collected from veterinary farm, University of Nigeria, Nsukka.

Characteristics of the wastes used in this study

The slurry of plantain/almond leaves and algae from sewage pond (P/A and A) was obtained by diluting the solid wastes with water in the mass ratio of 1 : 4 (waste : water). This implies that a total of 23 kg of plantain/almond leaves and algae (P/A and A) from sewage pond was mixed with 92 kg of water giving a total mass of 115 kg of slurry. These were measured using a weighing balance of 0 to 50

kg ranges. Both waste and water were thoroughly mixed in a small drum ensuring that no solid (hard) material, which was not decomposable, was present before introducing the mixture into the digester. Due to the high lignin which is non-degradable material, the waste (leaves) was seeded with algae water from sewage (inoculums) which boosted the rate of gas production. The waste

Table 1. Ratio and the temperature of the samples.

Waste	Mixing ratio	Quantity of waste and water (kg)	Ambient temperature range (°C)	Slurry temp range (°C)	Total volume biogas produced (L)
P/A and A	1 : 4	115	27.0 - 57.0	33.0 - 38.5	220.5
Pig dung	1 : 2	120	26.0 - 31.0	31.5 - 38.0	882.5

occupied about 74% by volume of the digester, this is the loading rate. The remaining part was left for gas collection. After introducing the waste, all openings were closed. After 1 day from charging, biogas generation commenced. The gas became combustible from the sixteenth day to the end of digestion. The total volume of gas produced was 96.0 L. The biogas became combustible as from the fourth day to the end of digestion as show in Table 1.

The initially dry pig dung was pulverized and every hard stones were removed. It was dissolved in water in the ratio of 1 : 2. A total of 40 kg of waste was mixed with 80 kg of water in batches in a small drum after which it was introduced into the digester (Figure 2) while keeping gas outlet open to exhaust trapped air. The fermentation and biogas production started after 1 day. The gas became combustible from the fourteenth day to the end of digestion. Batch operation was adopted.

RESULTS

Proximate analysis

The sample were analyzed for Ash, PH, total solid, volatile solid, moisture, phosphorus, fiber contents using the method of Association of Official and Analytical Chemistry (Sharma, 2002). Protein, fat contents were determined using Micro-Kjeldahl method. Carbon content was determined by the method of Walkey and Polpraset (Polpraset and Bitton, 1989 ; Aubart and Farinet, 1983).

The growth and catabolism of microbes need various kinds of nutrients especially elements of carbon, nitrogen and phosphorus. For high quality of methane, carbon is required for building of the cell structure of the methanogenic bacteria. Specific group of bacteria always consume carbon and nitrogen elements in a fixed proportion. From Tables 3 and 4, it was discovered that the value of protein/nitrogen, volatile solid, total solid and carbon in both samples decreased in percentage after digestion. Some of them were used up by the bacteria. The percentage of phosphorous was increased at end of digestion. The biofertilizer was rich with phosphorous and nitrogen which produces better yield when apply on farm land.

Storage of biogas

A gas compressor is a mechanical device that increases the pressure of a gas by reducing its volume. The capacity of the compressor used was 1/5 horse-power. Each cylinder was able to compress biogas of 1.2 bars of pressure. The biogas from pig dung and A/P and A

became combustible on the fourteenth and sixteenth days respectively and it burned with blue flame Each sample of biogas produced was analyzed using Orsat apparatus. The measuring principle of Orsat apparatus is the measurement of the reduction which occurs when individual constituents of a gas are removed separately by absorption in liquid reagents (Ezekoye and Okeke, 2006).

The daily ambient temperature and slurry temperature for almond/plantain leaves with algae and pig dung were shown in Figures 3 and 4. Almond/plantain leaves with algae recorded the highest temperature range of 27.0 – 37.0°C and the 2 wastes produced biogas within the mesophilic range of temperature (Itodo and Philips, 2002; Goodger, 1980). Orsat apparatus was used to analysis the gas produced. The methane component of gas from pig dung was 70.2% and for almond/plantain leaves with algae was 72.7%.

The daily volumes of biogas yield of the almond/plantain leaves with algae and pig dung were shown in Figure 5. Careful examination of the curves shows that the pig dung generated the highest gas from the first day to the thirty-seventh day. It was followed by almond/plantain leaves with algae which produced the highest between twelfth and thirteenth days.

On the other hand, the pig dung slurry produced combustible gas on the fourteenth day and almond/plantain leaves with algae (A/P and A) the sixteenth day (Table 2). The cumulative biogas yields of the sample are compared in Figure 6. The pig dung gave the highest yield 882.5 L. The almond/plantain leaves with algae produced 220.5 L.

DISCUSSION

For optimum functioning, the anaerobic micro-organisms require a neutral environment. The optimal pH was found to be 6.00 to 7.5 for pig dung, 6.00 to 7.3 for almond/plantain leaves with algae (Tables 3 and 4). Both acid and methane forming bacteria could not survive the pH values of 4 and 10.

Different wastes were used in feeding the digester to find out which one produced more gas. It was found that organic waste which is easily digestible produced more gas. Material with high lignocellulose produces less amount of gas. Carbon, which constitutes the basic frame of all organic substrates, provides microbes with the energy required for their living activities, and is the source

Table 2. Days of flammability and total biogas produced.

Waste	Flammable time (day)	Retention time (days)	Total biogas produced (L)
P/A&A	16	43	220.5
Pig dung	14	43	882.5

Table 3. Proximate analysis for almond/plantain leaves and algae.

Components	Before digestion	After digestion
Protein	1.62%	1.31%
Fats	2.05%	0.86%
pH	6.50	7.30
Moisture	89.32%	65.97%
Ash	0.50%	2.0%
Fiber	5.75%	Trace
Carbohydrate	24.12%	6.50%
Total solid (TS)	14.83% ¹	13.00%
Volatile solid (VS)	11.50%	6.67%
Carbon	0.29%	0.19%
Phosphorus	1.38ppm	7.71ppm

PPM; Part per million.

Table 4. Proximate analysis for pig dung.

Components	Before digestion	After digestion
Protein	6.63%	1.06%
Fats	0.78%	1.33%
pH	6.00	7.50
Moisture	85.67%	79.92%
Ash	1.90%	0.44%
Fiber	Trace	Trace
Carbohydrate	10.59%	11.66%
Total solid (TS)	16.00%	12.70%
Volatile solid (VS)	7.70%	5.00%
Carbon	1.09%	0.21%
Phosphorus	3.93ppm	5.76ppm

PPM; Part per million.

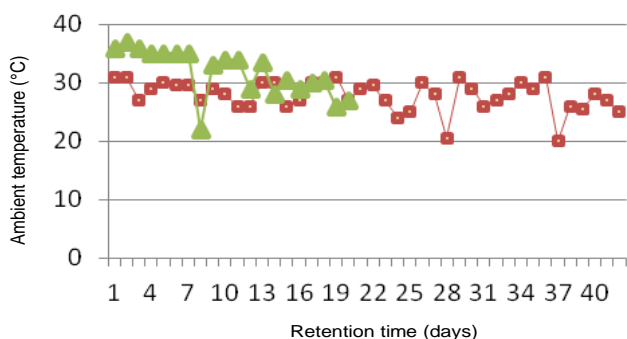


Figure 3. Change in ambient temperature during fermentation.

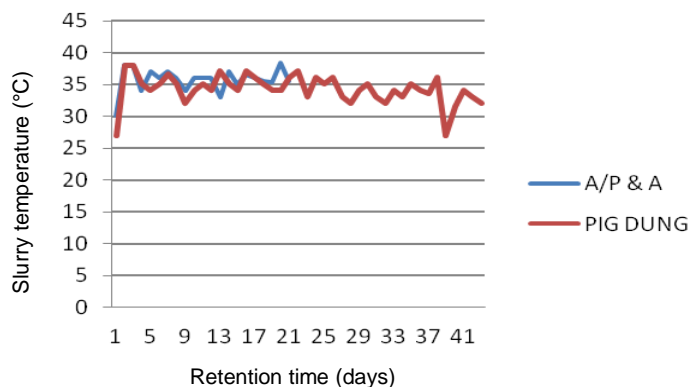


Figure 4. Change in slurry temperature during fermentation.

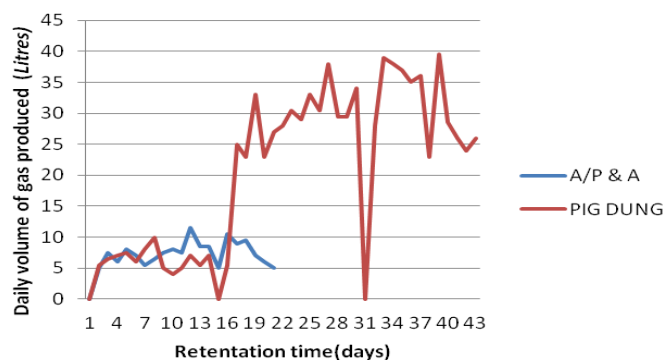


Figure 5. Volume of gas produced by A/P and A and pig dung leaves during fermentation.

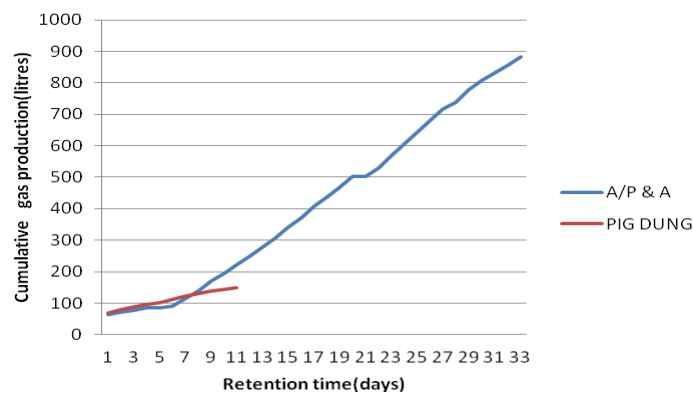


Figure 6. Daily cumulative gas produced by almond/plantain leaves.

Table 5. Percentage of the component of biogas from 2 different wastes using Orsat apparatus.

Waste	Carbon dioxide (CO ₂) (%)	Hydrogen sulphide H ₂ S (%)	Carbon monoxide (CO) (%)	Methane and other components (%)
Pig dung	24.9	1.2	0.5	70.2
A/P&A	17.5	0.8	9.0	72.7

for the formation of biogas. In biogas production, nitrogen provides methanogenic bacteria with ammonia, which is the source of nitrogen for the composition of living matter of new cells. The carbon/nitrogen ratio for pig dung was 8 : 1, for almond/plantain leaves with algae, it was 6 : 1

The slurry should not be too thick nor too dilute. In this experiment the dilution ratios used for pig dung and Almond/Plantain leaves with Algae were 1 : 2 and 1 : 4 respectively. Stirring is necessary for increased gas production. Gas production was found to be low at pH 4 and 9. When the slurry was stirred once in a day, there was an increase in gas production. There was also a drop in gas production, when stirring was completely omitted due to scum formation. The slurry was seeded with bacteria (algae from sewage pond). After the analysis of the slurry, it was discovered that there was an increase in the percentage content of nitrogen, potassium, protein and phosphorus (N P K) after digestion. This shows that the sludge is a better fertilizer to the soil (Tables 3 and 4) (Chemical Land 21, 2006). The importance of inoculum is that it fastened the establishment of anaerobic micro flora and thus eliminates the unnecessary lag phase observed during the start-up.

Table 2 shows that pig dung gives the shortest flammability time of fourteenth (14th) day followed by A/P and A sixteenth (16th) day. This shows that the day the biogas started burning should be in the order: pig dung < A/P and A.

Figure 7 shows the cumulative biogas generation during the fermentation. This connotes that the biogas yield should be in the order: pig dung > A/P and A. This means that pig dung produced the highest biogas. Table 5 shows the composition of biogas produced. Biogas generation has the following applications: production of energy (heat, light, electricity); transformation of organic wastes into high quality fertilizer; reduction of workload, mainly for women, in firewood collection and cooking; and environmental advantages through protection of forests, soil, water and air.

Conclusion

The result of this study showed that almond leaves, plantain leaves, pig dung, and algae from sewage which can cause pollution in the environment can be a useful source of energy by subjecting it to anaerobic digestion for biogas production. The microorganism associated

with the fermentation of plantain leaves/almond leaves mainly originated from the inoculum used. The addition of inoculums (algae from sewage pond) to the leaves (almond/plantain) was found to enhance gas production. The pig dung anaerobically digested showed the highest yield for methane production. It is also expected that this will be a source of waste management and pollution control.

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